Topology Prelim, Summer 2006

Do 8 of the 12 problems. Use separate sheets for different problems. Include a cover sheet which lists the eight problems that you chose to do.

1. Prove that the continuous image of a compact topological space is compact.

2. Let Y and Z be two closed subsets of a space X such that $X = Y \cup Z$, and let $f : X \to X$. Prove that f is continuous iff both $f|Y$ and $f|Z$ are continuous.

3. Let X be a compact Hausdorff space. Suppose C_n, for $n = 0, 1, 2, \ldots$ is a collection of closed and connected subsets of X such that $C_{n+1} \subseteq C_n$. Prove that $\bigcap_{n=0}^{\infty} C_n$ is connected.

4. Prove that the product of two regular spaces is regular.

5. Prove that every continuous bijection from a compact space to a Hausdorff space is a homeomorphism.

6. Suppose A and B are two disjoint compact sets in a Hausdorff space (X, T). Show that there are disjoint open sets U containing A, and V containing B.

7. Suppose (X, T) and (X', T') are two topological spaces with (X', T') Hausdorff, f and g are continuous functions from X to X', and K is the subset $\{x : f(x) = g(x)\}$ of X. Show that K is closed in X.

8. Let X be the set of real numbers, and define a new topology T' on X by defining a set V to be open in T' iff it is of the form $U \setminus C$, where U is open in the usual Euclidean topology, and C is countable. Determine which separation properties T_n hold in X, $n = 0, 1, 2, 3, 4$.

9. Determine, with explanation, which two of the ~~two~~ three pictured subsets of the plane have the same fundamental group.

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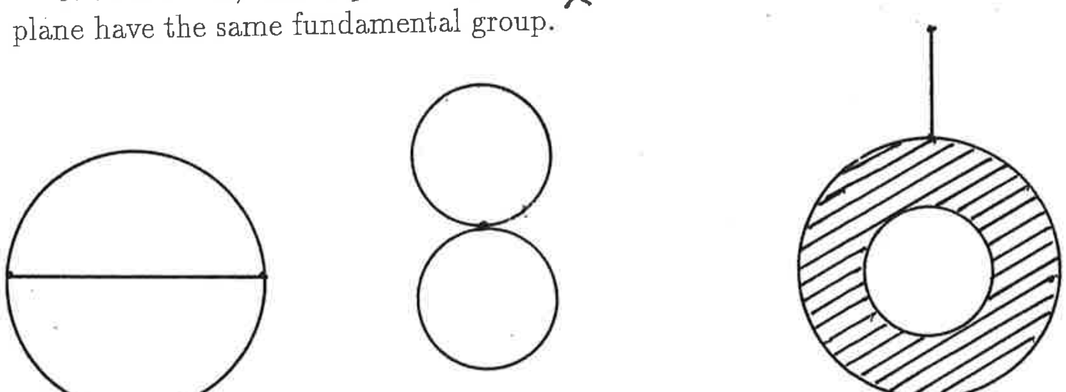

10. Define what it means to say that the space (X,T) is contractible, then show that every contractible space is path connected.

11. Suppose (X'',T''), (X,T) and (X',T') are three topological spaces, p is a covering map from X'' onto X, and q is a covering map from X onto X' such that for each point y in X', the inverse of y under q is finite. Show that the composition $q \circ p$ is a covering map.

12. Find a nontrivial (i.e., connected and not homeomorphic to the original space) covering space for the pictured subset of the plane.