

DISCOUNTED CASH FLOW  
VALUATION

SUPPOSE YOU HAVE \$100 TODAY...

## FUTURE VALUE DEFINITIONS

- Future Value (FV): The amount an investment is worth after one or more periods.
- Simple Interest: Interest earned only on the original principal amount invested.
- Compound Interest: Interest earned on both the initial principal and the interest reinvested from prior periods.

## FUTURE VALUE

- Compounding: The process of accumulating interest on an investment over time to earn more interest.
- Future Value:

$$FV_t = PV_0 \times (1 + r)^t$$

## FUTURE VALUE: EXAMPLE #1

You deposit \$500 into a savings account. You plan on withdrawing the money and closing the account exactly two years from today. Interest rates are 10%, compounded annually, and will remain constant over the two years.

## FUTURE VALUE: EXAMPLE #1

- How much money will you have when you close the account (future value)?
- How much simple interest did you accumulate?
- How much compound interest did you accumulate?

## THE EFFECTS OF COMPOUNDING

- The effects/benefits of compounding:
  - Increase with the interest rate.
  - Increase with time.
  - Increase with the frequency of compounding.  
*(more on the details of this later)*

## FUTURE VALUE: EXAMPLE #2

- You are scheduled to receive \$17,000 in two years. When you receive it, you will invest it for six more years at 6 percent per year. How much will you have eight years from today?

## FUTURE VALUE: EXAMPLE #3

- You are trying to save to put a \$40,000 down payment on a home. You have \$20,000 today that can be invested at your bank. The bank pays 4 percent annual interest on its accounts. How long will it be before you have enough?

## FUTURE VALUE: EXAMPLE #4

- Assume you are only willing to wait 10 years in the previous example. What rate of return would you need to earn?

## PRESENT VALUE DEFINITIONS

- Present Value (PV): The current value of future cash flows discounted at the appropriate discount rate.
- Discount: Calculate the present value of some future amount.
- Discount Rate: The rate used to calculate the present value of future cash flows.

## CALCULATING PRESENT VALUE

- Future Value:

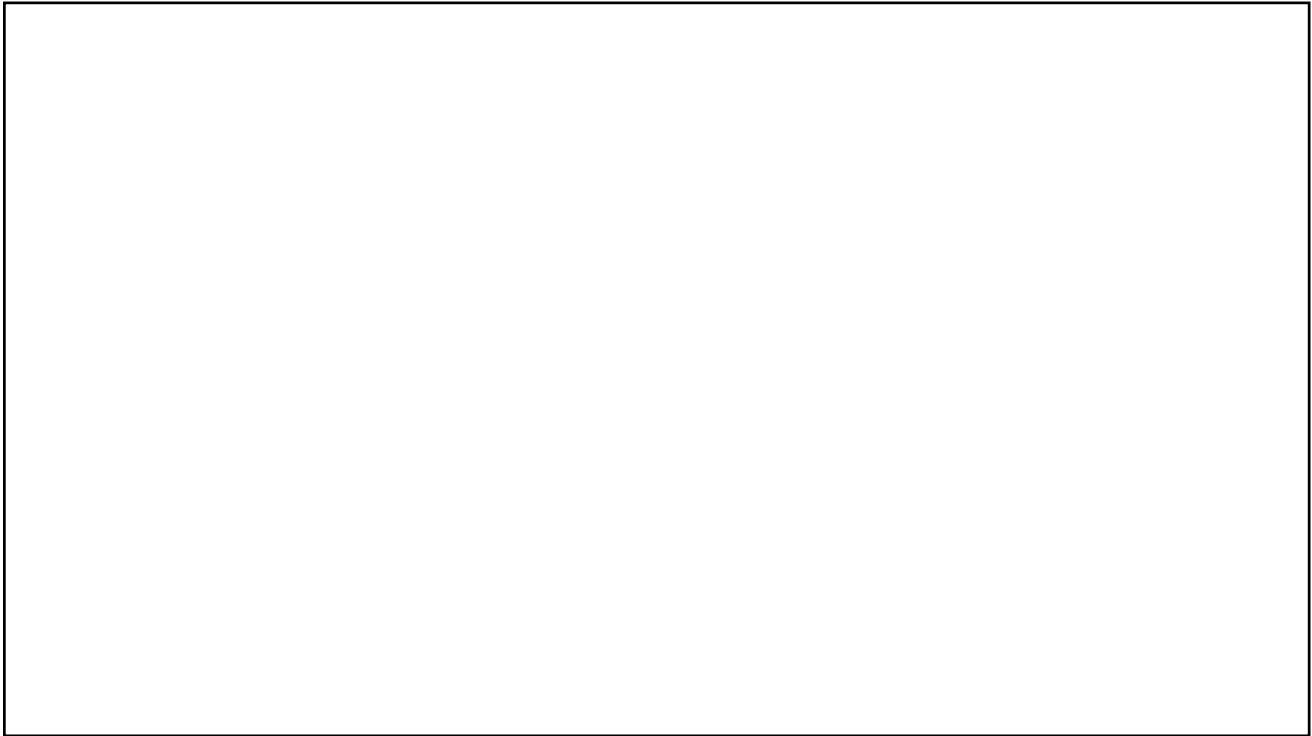
$$FV_t = PV_0 \times (1 + r)^t$$

- Present Value:

$$PV_0 = \frac{FV_t}{(1 + r)^t}$$

## PRESENT VALUE: EXAMPLE #1

- You have five of the six Florida Lottery numbers. Lottery officials offer you the choice of the following alternative payouts:
  - Alternative 1: \$100,000 one year from now.
  - Alternative 2: \$200,000 five years from now.
- Which alternative would you choose if interest rates are 12%?



What rate makes the two alternatives equally attractive?



## PRESENT VALUE: EXAMPLE #2

- Suppose you are still committed to a \$40,000 down payment. If you believe your mutual fund can achieve a 9 percent annual rate of return and you want to buy the house in 5 years, how much must you invest today?

## TIPS ON SOLVING PRESENT VALUE AND FUTURE VALUE PROBLEMS

- For multiple cash flows, just add up the individual present (or future) values.
  - As  $t \uparrow$ ,  $PV \downarrow$  and  $FV \uparrow$
  - As  $r \uparrow$ ,  $PV \downarrow$  and  $FV \uparrow$
- There are (currently) only 4 components: PV, FV,  $t$ , and  $r$ 
  - With ANY 3 components, you can solve for the 4<sup>th</sup>

## FINANCIAL CALCULATORS

### REVISITING PRESENT VALUE: EXAMPLE #2

- Suppose you are still committed to a \$40,000 down payment. If you believe your mutual fund can achieve a 9 percent annual rate of return and you want to buy the house in 5 years, how much must you invest today?

REVISITING  
FUTURE VALUE: EXAMPLE #2

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## REVISITING FUTURE VALUE: EXAMPLE #4

- Assume you are only willing to wait 10 years in the previous example. What rate of return would you need to earn?

## PRESENT AND FUTURE VALUE OF MULTIPLE CASH FLOWS

$$FV_t = CF_0 \times (1 + r)^t + CF_1 \times (1 + r)^{t-1} + \dots + CF_t$$

$$PV_0 = CF_0 + \frac{CF_1}{(1 + r)^1} + \frac{CF_2}{(1 + r)^2} + \dots + \frac{CF_t}{(1 + r)^t}$$

- You just inherited some money from now dead Uncle Fred. You plan to use the money for a vacation, but know you first need to put aside some to cover your books and supplies over the next two years. You expect to need \$4,000 in each of the next two years. Interest rates are 10%, compounded annually. How much of now dead Uncle Fred's money do you need to put aside today?

## VALUING PERPETUITIES

- Perpetuity: A level stream of cash flows which continue forever (sometimes called consols).
- Present Value of a Perpetuity:

$$PV_0 = \frac{CF_1}{r}$$

## VALUING PERPETUITIES

- Assuming that interest rates are 10%, what is the value today of a perpetuity paying \$500 per year, with the first payment one year from today?

## VALUING PERPETUITIES

- Would you be willing to pay \$6,500 for the same perpetuity if interest rates were 8%?

## GROWING PERPETUITIES

- Present Value of a Growing Perpetuity:

$$PV_0 = \frac{CF_1}{r - g}$$

## GROWING PERPETUITIES

- Assume a growing perpetuity just made a payment of \$120 yesterday. If the cash flow is expected to grow at 5% and interest rates are still 10%, what is the price of the perpetuity today?


## PRESENT VALUE OF AN ANNUITY

- Annuity: A level stream of cash flows for a fixed period of time.
- Present Value of an Annuity:

$$PV_0 = \frac{CF_1}{r} \times \left[ 1 - \frac{1}{(1+r)^t} \right]$$

## PRESENT VALUE OF AN ANNUITY

- We can rearrange the equation to the following:
- Present Value of an Annuity:

$$PV_0 = \frac{CF_1}{r} \times \left[ 1 - \frac{1}{(1+r)^t} \right] \quad PV_0 = CF_1 \times \frac{\left[ 1 - \frac{1}{(1+r)^t} \right]}{r}$$




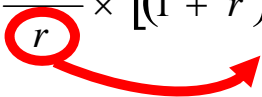
## PRESENT VALUE OF AN ANNUITY

*Let's return to our earlier example:*

- You just inherited some money from now dead Uncle Fred. You plan to use the money for a vacation, but know you first need to put aside some to cover your books and supplies over the next two years. You expect to need \$4,000 in each of the next two years. Interest rates are 10%. How much of now dead Uncle Fred's money do you need to put aside today?

## FUTURE VALUE OF AN ANNUITY

- Future Value of an Annuity:

$$FV_t = \frac{CF}{r} \times [(1 + r)^t - 1]$$


- This can also be rearranged to...

$$FV_t = CF \times \frac{[(1 + r)^t - 1]}{r}$$

## FUTURE VALUE OF AN ANNUITY

- If you deposit \$300 into a retirement account at the end of each month, starting next month, and the account earns 0.75% per month, how much will you have in 35 years?

## ANNUITIES: A REAL-LIFE EXAMPLE

- Books and beer are expensive! You now have a balance of \$2,000 on your VISA card. The interest rate on that card is 2% per month. However, you pay only the \$50 minimum payment each month (starting next month) and make no more charges on that card. How long will it take you to pay off the balance?

## GROWING ANNUITIES

- Present Value of a Growing Annuity:

$$PV_0 = \frac{CF_1}{r - g} \times \left[ 1 - \left( \frac{1 + g}{1 + r} \right)^t \right]$$

## ANNUITIES DUE

- Annuity Due: An annuity for which the cash flows occur at the *beginning* of the period.

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- PV Annuity Due  
= (PV Ordinary Annuity) x (1 + r)
- FV Annuity Due  
= (FV Ordinary Annuity) x (1 + r)

## Discounted Cash Flow Valuation

Annual Percentage Rate (APR): The nominal, stated annual interest rate that ignores the effect of compound interest within the year. The APR is the periodic rate ( $r$ ) times the number of compoundings per year ( $m$ ).

12% APR compounded quarterly

Annual Percentage Rate (APR): The nominal, stated annual interest rate that ignores the effect of compound interest within the year. The APR is the periodic rate ( $r$ ) times the number of compoundings per year ( $m$ ).

12% APR compounded monthly =

12% APR compounded quarterly =

12% APR compounded semiannually =

12% APR compounded annually =

## THE EFFECT OF COMPOUNDING

- Effective Annual Rate (EAR): The effective annual interest rate, which takes into account the effect of compound interest.

## APR AND EAR

- Example: A bank loan is quoted as 12% APR, compounded semiannually. What is the EAR?

## APR AND EAR

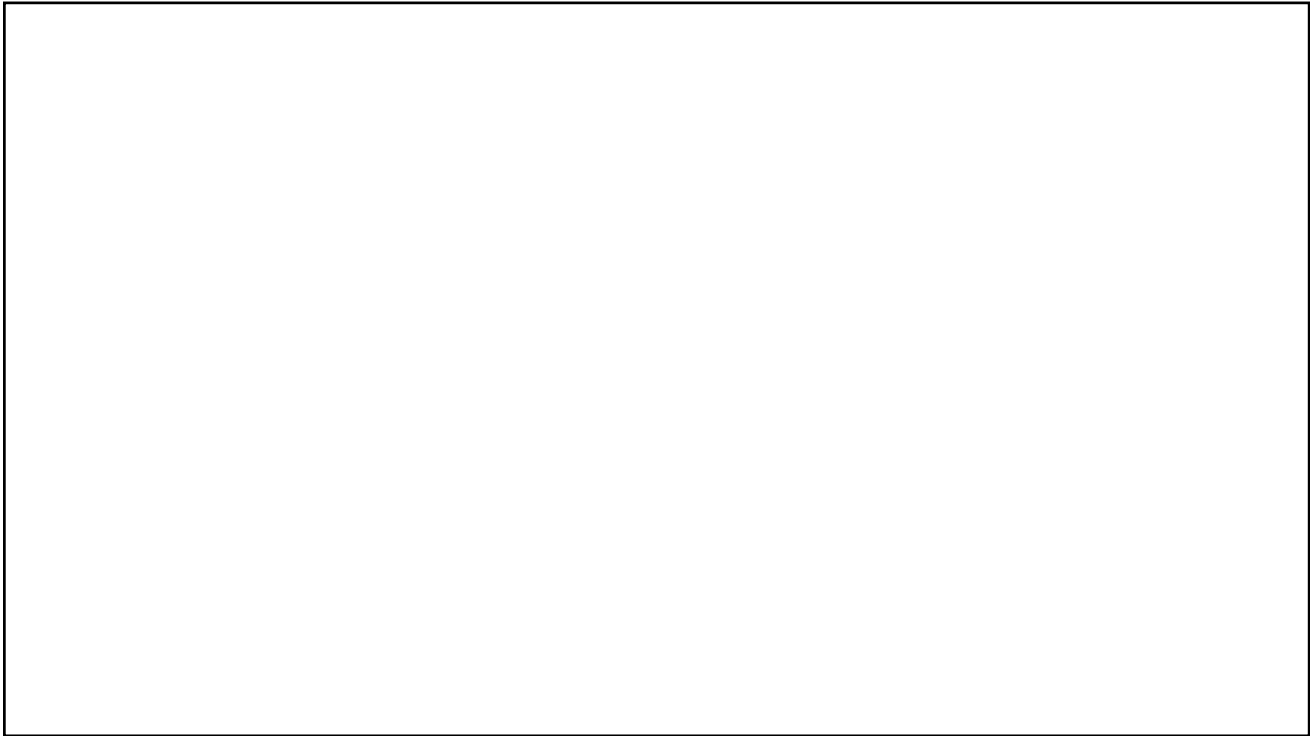
- Example: A bank loan is quoted as 12% APR, compounded semiannually. What is the EAR?

$$EAR = \left[ 1 + \left( \frac{APR}{m} \right) \right]^m - 1$$

## AMORTIZATION

- What is an amortized loan?
- You plan to buy a \$200,000 house. You will put 10% down and finance the rest with a 30 year mortgage at 6% APR, compounded monthly. What are the monthly payments?

## Discounted Cash Flow Valuation



### AMORTIZATION SCHEDULE

Month	Beg. Bal	PMT	Interest	Principal	End. Bal.
1					
2					
3					
4					
357	4,263.34	1,079.19	21.32	1,057.87	3,205.46
358	3,205.46	1,079.19	16.03	1,063.16	2,142.30
359	2,142.30	1,079.19	10.71	1,068.48	1,073.82
360	1,073.82	1,079.19	5.37	1,073.82	0.00



## CHAPTER 4 SUGGESTED PROBLEMS

- Concept Questions
  - 1 through 6, and 8
- Questions and Problems
  - 1 through 7, 9, 11, 12, 15, 19, 23, 24, 30, 35, 36, 41, 43, 49, and 52

## ADDITIONAL PRACTICE

- Assuming a 10% interest rate, compounded annually, what is the value today of \$1,000 per year forever, with the first payment starting one year from today?

## ADDITIONAL PRACTICE

- What if the first payment was in 5 years?

## ADDITIONAL PRACTICE

- Given an interest rate of 10% APR, compounded annually, what is the value in five years of a perpetual stream of \$120 annual payments starting in nine years?

## ADDITIONAL PRACTICE

- You have just read an advertisement that says, “Pay us \$100 a year for 10 years, starting next year, and we will pay you (and your heirs) \$100 a year thereafter in perpetuity.” At what range of interest rates would you accept this deal?