DISCOUNTED CASH FLOW VALUATION

SUPPOSE YOU HAVE $100 TODAY…

FUTURE VALUE DEFINITIONS

- *Future Value (FV)*: The amount an investment is worth after one or more periods.
- *Simple Interest*: Interest earned only on the original principal amount invested.
- *Compound Interest*: Interest earned on both the initial principal and the interest reinvested from prior periods.

FUTURE VALUE

- *Compounding*: The process of accumulating interest on an investment over time to earn more interest.
- Future Value:

$$FV_t = PV_0 \times (1 + r)^t$$

FUTURE VALUE: EXAMPLE #1

You deposit $500 into a savings account. You plan on withdrawing the money and closing the account exactly two years from today. Interest rates are 10%, compounded annually, and will remain constant over the two years.

FUTURE VALUE: EXAMPLE #1

- How much money will you have when you close the account (future value)?
- How much simple interest did you accumulate?
- How much compound interest did you accumulate?

THE EFFECTS OF COMPOUNDING

- The effects/benefits of compounding:
  - Increase with the interest rate.
  - Increase with time.
  - Increase with the frequency of compounding.

(more on the details of this later)

FUTURE VALUE: EXAMPLE #2

- You are scheduled to receive $17,000 in two years. When you receive it, you will invest it for six more years at 6 percent per year. How much will you have eight years from today?

FUTURE VALUE: EXAMPLE #3

- You are trying to save to put a $40,000 down payment on a home. You have $20,000 today that can be invested at your bank. The bank pays 4 percent annual interest on its accounts. How long will it be before you have enough?

FUTURE VALUE: EXAMPLE #4

- Assume you are only willing to wait 10 years in the previous example. What rate of return would you need to earn?

PRESENT VALUE DEFINITIONS

- *Present Value (PV)*: The current value of future cash flows discounted at the appropriate discount rate.
- *Discount*: Calculate the present value of some future amount.
- *Discount Rate*: The rate used to calculate the present value of future cash flows.

CALCULATING PRESENT VALUE

- Future Value:

$$FV_t = PV_0 \times (1 + r)^t$$

- Present Value:

$$PV_0 = \frac{FV_t}{(1 + r)^t}$$

PRESENT VALUE: EXAMPLE #1

- You have five of the six Florida Lottery numbers. Lottery officials offer you the choice of the following alternative payouts:
  - Alternative 1: $100,000 one year from now.
  - Alternative 2: $200,000 five years from now.
- Which alternative would you choose if interest rates are 12%?

What rate makes the two alternatives equally attractive?

PRESENT VALUE: EXAMPLE #2

- Suppose you are still committed to a $40,000 down payment. If you believe your mutual fund can achieve a 9 percent annual rate of return and you want to buy the house in 5 years, how much must you invest today?

TIPS ON SOLVING PRESENT VALUE AND FUTURE VALUE PROBLEMS

- For multiple cash flows, just add up the individual present (or future) values.
  - As t ↑, PV ↓ and FV ↑
  - As r ↑, PV ↓ and FV ↑
  - There are (currently) only 4 components: PV, FV, t, and r
    - With ANY 3 components, you can solve for the 4th

FINANCIAL CALCULATORS

REVISITING PRESENT VALUE: EXAMPLE #2

- Suppose you are still committed to a $40,000 down payment. If you believe your mutual fund can achieve a 9 percent annual rate of return and you want to buy the house in 5 years, how much must you invest today?

REVISITING FUTURE VALUE: EXAMPLE #2

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REVISITING FUTURE VALUE: EXAMPLE #3

- You are trying to save to put a $40,000 down payment on a home. You have $20,000 today that can be invested at your bank. The bank pays 4 percent annual interest on its accounts. How long will it be before you have enough?

REVISITING FUTURE VALUE: EXAMPLE #4

- Assume you are only willing to wait 10 years in the previous example. What rate of return would you need to earn?

PRESENT AND FUTURE VALUE OF MULTIPLE CASH FLOWS

$$FV_t = CF_0 \times (1 + r)^t + CF_1 \times (1 + r)^{t-1} + \ldots + CF_t$$

$$PV_0 = CF_0 + \frac{CF_1}{(1 + r)^1} + \frac{CF_2}{(1 + r)^2} + \ldots + \frac{CF_t}{(1 + r)^t}$$

- You just inherited some money from now dead Uncle Fred. You plan to use the money for a vacation, but know you first need to put aside some to cover your books and supplies over the next two years. You expect to need $4,000 in each of the next two years. Interest rates are 10%, compounded annually. How much of now dead Uncle Fred's money do you need to put aside today?

VALUING PERPETUITIES

- Perpetuity: A level stream of cash flows which continue forever (sometimes called consols).
- Present Value of a Perpetuity:

$$PV_0 = \frac{CF_1}{r}$$

VALUING PERPETUITIES

- Assuming that interest rates are 10%, what is the value today of a perpetuity paying $500 per year, with the first payment one year from today?

VALUING PERPETUITIES

- Would you be willing to pay $6,500 for the same perpetuity if interest rates were 8%?

GROWING PERPETUITIES

- Present Value of a Growing Perpetuity:

$$PV_0 = \frac{CF_1}{r - g}$$

GROWING PERPETUITIES

- Assume a growing perpetuity just made a payment of $120 yesterday. If the cash flow is expected to grow at 5% and interest rates are still 10%, what is the price of the perpetuity today?

PRESENT VALUE OF AN ANNUITY

- Annuity: A level stream of cash flows for a fixed period of time.
- Present Value of an Annuity:

$$PV_0 = \frac{CF_1}{r} \times \left[1 - \frac{1}{(1+r)^t}\right]$$

PRESENT VALUE OF AN ANNUITY

- We can rearrange the equation to the following:
- Present Value of an Annuity:

$$PV_0 = \frac{CF_1}{r} \times \left[1 - \frac{1}{(1+r)^t}\right] \qquad PV_0 = CF_1 \times \frac{\left[1 - \frac{1}{(1+r)^t}\right]}{r}$$

PRESENT VALUE OF AN ANNUITY

Let's return to our earlier example:

- You just inherited some money from now dead Uncle Fred. You plan to use the money for a vacation, but know you first need to put aside some to cover your books and supplies over the next two years. You expect to need $4,000 in each of the next two years. Interest rates are 10%. How much of now dead Uncle Fred's money do you need to put aside today?

FUTURE VALUE OF AN ANNUITY

- Future Value of an Annuity:

$$FV_t = \frac{CF}{r} \times \left[(1 + r)^t - 1\right]$$

- This can also be rearranged to…

$$FV_t = CF \times \frac{\left[(1 + r)^t - 1\right]}{r}$$

FUTURE VALUE OF AN ANNUITY

- If you deposit $300 into a retirement account at the end of each month, starting next month, and the account earns 0.75% per month, how much will you have in 35 years?

ANNUITIES: A REAL-LIFE EXAMPLE

- Books and beer are expensive! You now have a balance of $2,000 on your VISA card. The interest rate on that card is 2% per month. However, you pay only the $50 minimum payment each month (starting next month) and make no more charges on that card. How long will it take you to pay off the balance?

GROWING ANNUITIES

- Present Value of a Growing Annuity:

$$PV_0 = \frac{CF_1}{r - g} \times \left[1 - \left(\frac{1 + g}{1 + r} \right)^t \right]$$

ANNUITIES DUE

- Annuity Due: An annuity for which the cash flows occur at the *beginning* of the period.

ANNUITIES DUE

- Annuity Due: An annuity for which the cash flows occur at the *beginning* of the period.
- PV Annuity Due
  = (PV Ordinary Annuity) x (1 + r)
- FV Annuity Due
  = (FV Ordinary Annuity) x (1 + r)

Annual Percentage Rate (APR): The nominal, stated annual interest rate that ignores the effect of compound interest within the year. The APR is the periodic rate (r) times the number of compoundings per year (m).

12% APR compounded quarterly

Annual Percentage Rate (APR): The nominal, stated annual interest rate that ignores the effect of compound interest within the year. The APR is the periodic rate (r) times the number of compoundings per year (m).

12% APR compounded monthly =

12% APR compounded quarterly =

12% APR compounded semiannually =

12% APR compounded annually =

THE EFFECT OF COMPOUNDING

- Effective Annual Rate (EAR): The effective annual interest rate, which takes into account the effect of compound interest.

APR AND EAR

- Example: A bank loan is quoted as 12% APR, compounded semiannually. What is the EAR?

APR AND EAR

- Example: A bank loan is quoted as 12% APR, compounded semiannually. What is the EAR?

$$EAR = \left[1 + \left(\frac{APR}{m}\right)\right]^{m} - 1$$

AMORTIZATION

- What is an amortized loan?

- You plan to buy a $200,000 house. You will put 10% down and finance the rest with a 30 year mortgage at 6% APR, compounded monthly. What are the monthly payments?

AMORTIZATION SCHEDULE

Month	Beg. Bal	PMT	Interest	Principal	End. Bal.
1					
2					
3					
4					
357	4,263.34	1,079.19	21.32	1,057.87	3,205.46
358	3,205.46	1,079.19	16.03	1,063.16	2,142.30
359	2,142.30	1,079.19	10.71	1,068.48	1,073.82
360	1,073.82	1,079.19	5.37	1,073.82	0.00

CHAPTER 4 SUGGESTED PROBLEMS

- Concept Questions
  - 1 through 6, and 8
- Questions and Problems
  - 1 through 7, 9, 11, 12, 15, 19, 23, 24, 30, 35, 36, 41, 43, 49, and 52

ADDITIONAL PRACTICE

- Assuming a 10% interest rate, compounded annually, what is the value today of $1,000 per year forever, with the first payment starting one year from today?

ADDITIONAL PRACTICE

- What if the first payment was in 5 years?

ADDITIONAL PRACTICE

- Given an interest rate of 10% APR, compounded annually, what is the value in five years of a perpetual stream of $120 annual payments starting in nine years?

ADDITIONAL PRACTICE

- You have just read an advertisement that says, “Pay us $100 a year for 10 years, starting next year, and we will pay you (and your heirs) $100 a year thereafter in perpetuity.” At what range of interest rates would you accept this deal?