Discounted Cash Flow Valuation

Future Value of Multiple Cash Flows

$$FV_t = CF_0 \times (1 + r)^t + CF_1 \times (1 + r)^{t-1} + \ldots + CF_t$$

- You open a bank account today with $500. You expect to deposit $1,000 at the end of each of the next three years. Interest rates are 5%, compounded annually. How much will you have in your account in three years?

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Present Value of Multiple Cash Flows

$$PV_0 = CF_0 + \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \ldots + \frac{CF_t}{(1+r)^t}$$

- You just inherited some money from now dead Uncle Fred. You plan to use the money for a vacation, but know you first need to put aside some to cover your books and supplies over the next two years. You expect to need $4,000 in each of the next two years. Interest rates are 10%, compounded annually. How much of now dead Uncle Fred's money do you need to put aside today?

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Valuing Perpetuities

- Perpetuity: A level stream of cash flows which continue forever (sometimes called consols).
- Present Value of a Perpetuity:

Valuing Perpetuities

- Assuming that interest rates are 10%, what is the value today of a perpetuity paying $500 per year, with the first payment one year from today?

Valuing Perpetuities

- Would you be willing to pay $6,500 for the same perpetuity if interest rates were 8%?

Growing Perpetuities

- Present Value of a Growing Perpetuity:

Growing Perpetuities

- Suppose you own a perpetuity that promises to pay $1 next year, after which the payment is expected to grow at 5% per year forever. If interest rates are 10%, what is the value of the perpetuity?

Growing Perpetuities

- Assume a growing perpetuity just made a payment of $120 yesterday. If the cash flow is expected to grow at 5% and interest rates are still 10%, what is the price of the perpetuity today?

Present Value of an Annuity

- Annuity: A level stream of cash flows for a fixed period of time.

- Present Value of an Annuity:

$$PV_0 = \frac{CF_1}{r} \times \left[1 - \frac{1}{(1+r)^t}\right]$$

Present Value of an Annuity

- We can rearrange the equation to the following:

- Present Value of an Annuity:

$$PV_0 = CF_1 \times \frac{\left[1 - \frac{1}{(1+r)^t}\right]}{r}$$

Present Value of an Annuity

Let's return to our earlier example:

- You just inherited some money from now dead Uncle Fred. You plan to use the money for a vacation, but know you first need to put aside some to cover your books and supplies over the next two years. You expect to need $4,000 in each of the next two years. Interest rates are 10%. How much of now dead Uncle Fred's money do you need to put aside today?

Future Value of an Annuity

- Future Value of an Annuity:

$$FV_t = \frac{CF}{r} \times \left[(1 + r)^t - 1 \right]$$

- This, of course, can also be rearranged…

$$FV_t = CF \times \frac{\left[(1 + r)^t - 1 \right]}{r}$$

Future Value of an Annuity

- What is the future value (at year 2) of the previous example?

Annuities: A Real-Life Example

- Books and beer are expensive! You now have a balance of $2,000 on your VISA card. The interest rate on that card is 2% per month. However, in an attempt to not let your debt stifle your social life, you pay only the $50 minimum payment each month (starting next month) and make no more charges on that card. How long will it take you to pay off the balance?

Annuities: A Real-Life Example

- How much would you have to pay each month if you wanted to pay off the balance in 3 years?

Growing Annuities

- Present Value of a Growing Annuity:

$$PV_0 = \frac{CF_1}{r - g} \times \left[1 - \left(\frac{1 + g}{1 + r} \right)^t \right]$$

Annuities Due

- Annuity Due: An annuity for which the cash flows occur at the _____ of the period.

Annuities Due

- Annuity Due: An annuity for which the cash flows occur at the *beginning* of the period.

- PV Annuity Due
  = (PV Ordinary Annuity) x (1 + r)

The Effect of Compounding

- Annual Percentage Rate (APR): The nominal, stated annual interest rate that ignores the effect of compound interest within the year. The APR is the periodic rate (r) times the number of compoundings per year (m).

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12% APR compounded quarterly

Annual Percentage Rate (APR): The nominal, stated annual interest rate that ignores the effect of compound interest within the year. The APR is the periodic rate (r) times the number of compoundings per year (m).

12% APR compounded monthly =
12% APR compounded quarterly =
12% APR compounded semiannually =
12% APR compounded annually =

The Effect of Compounding

- Effective Annual Rate (EAR): The effective annual interest rate, which takes into account the effect of compound interest.

APR and EAR

- Example: A bank loan is quoted as 10% APR, compounded semiannually. What is the EAR?

APR and EAR

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$$EAR = \left[1 + \left(\frac{APR}{m}\right)\right]^{m} - 1$$

APR and EAR: An Example

- Which loan would you choose?
  - Bank A: 15% compounded daily
  - Bank B: 15.5% compounded quarterly
  - Bank C: 16% compounded annually

Amortization

- What is an amortized loan?
- You plan to buy a $200,000 house. You will put 10% down and finance the rest with a 30 year mortgage at 6% APR, compounded monthly. What are the monthly payments?

Amortization Schedule

Month	Beg. Bal	PMT	Interest	Principal	End. Bal.
1					
2					
3					
4					
357	4,263.34	1,079.19	21.32	1,057.87	3,205.46
358	3,205.46	1,079.19	16.03	1,063.16	2,142.30
359	2,142.30	1,079.19	10.71	1,068.48	1,073.82
360	1,073.82	1,079.19	5.37	1,073.82	0.00

Chapter 6 Suggested Problems

- Concepts Review and Critical Thinking Questions:
  - 2 through 8
- Questions and Problems:
  - 1, 3, 4, 5, 7, 10, 12, 20, 21, 24, 26, 28, 36, 41, 43, 45, and 54

Additional Practice

- You want to buy a new, fully-loaded truck. You have managed to talk the salesman down to $40,000. You plan on putting a 10% down payment on it and have secured a 60 month loan at 9% APR, compounded monthly, for the balance. How much are your monthly payments?

Additional Practice

- Assuming a 10% interest rate, compounded annually, what is the value today of $1,000 per year forever, with the first payment starting one year from today?

Additional Practice

- What if the first payment was in 5 years?

Additional Practice

- Given an interest rate of 10% APR, compounded annually, what is the value in five years of a perpetual stream of $120 annual payments starting in nine years?

Additional Practice

- You have just read an advertisement that says, “Pay us $100 a year for 10 years, starting next year, and we will pay you (and your heirs) $100 a year thereafter in perpetuity.” At what range of interest rates would you accept this deal?