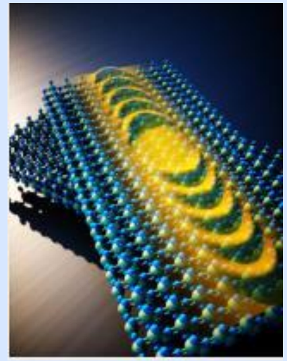
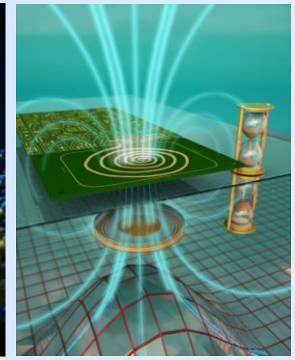




Extreme Light-Matter Interactions in Metamaterials

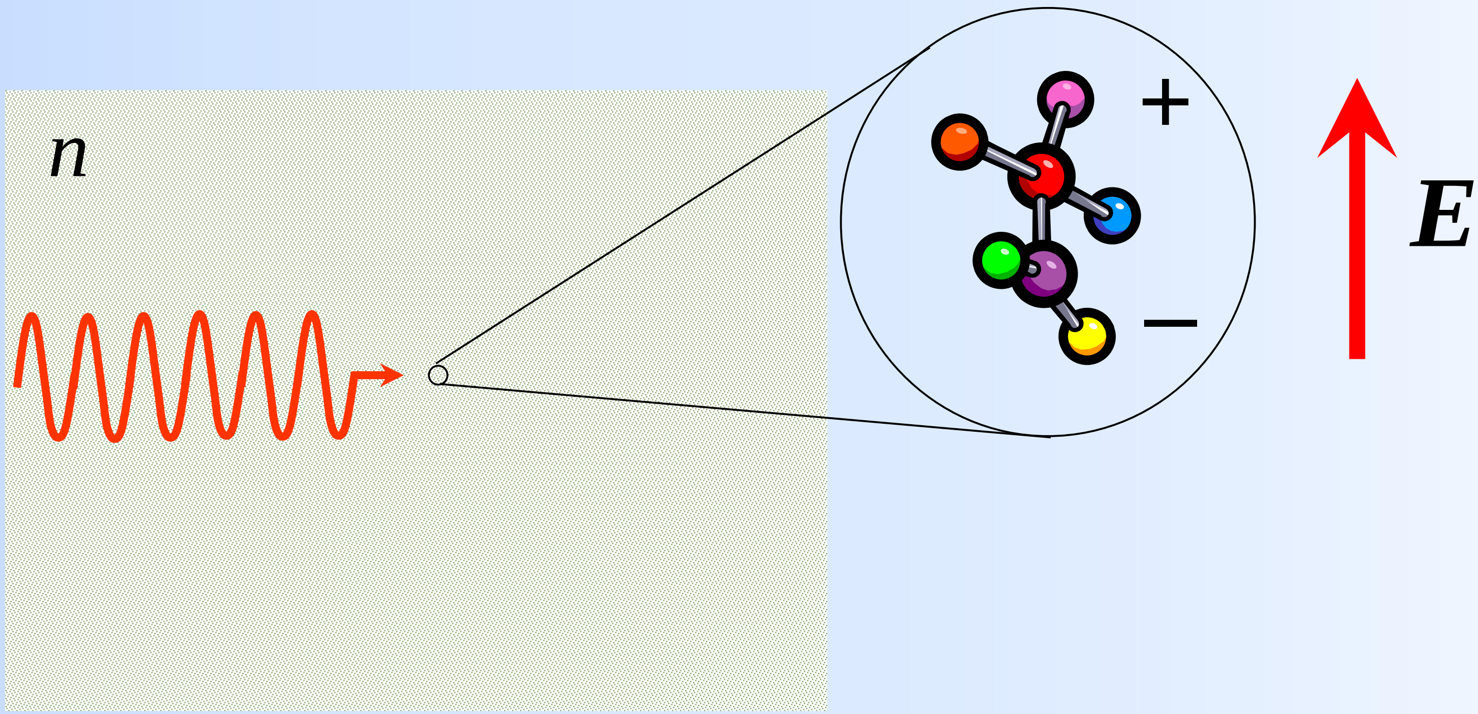
Andrea Alù

Photonics Initiative, Advanced Science Research Center, City University of New York
Physics Program, Graduate Center, City University of New York
Department of Electrical Engineering, City College of New York
http://alulab.org, aalu@gc.cuny.edu

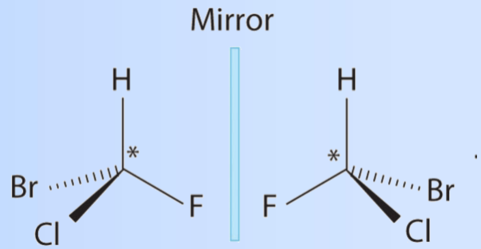
The City University of New York
ADVANCED SCIENCE RESEARCH CENTER

Metamaterials & Plasmonics Research Laboratory

Wave Interactions in Natural Materials

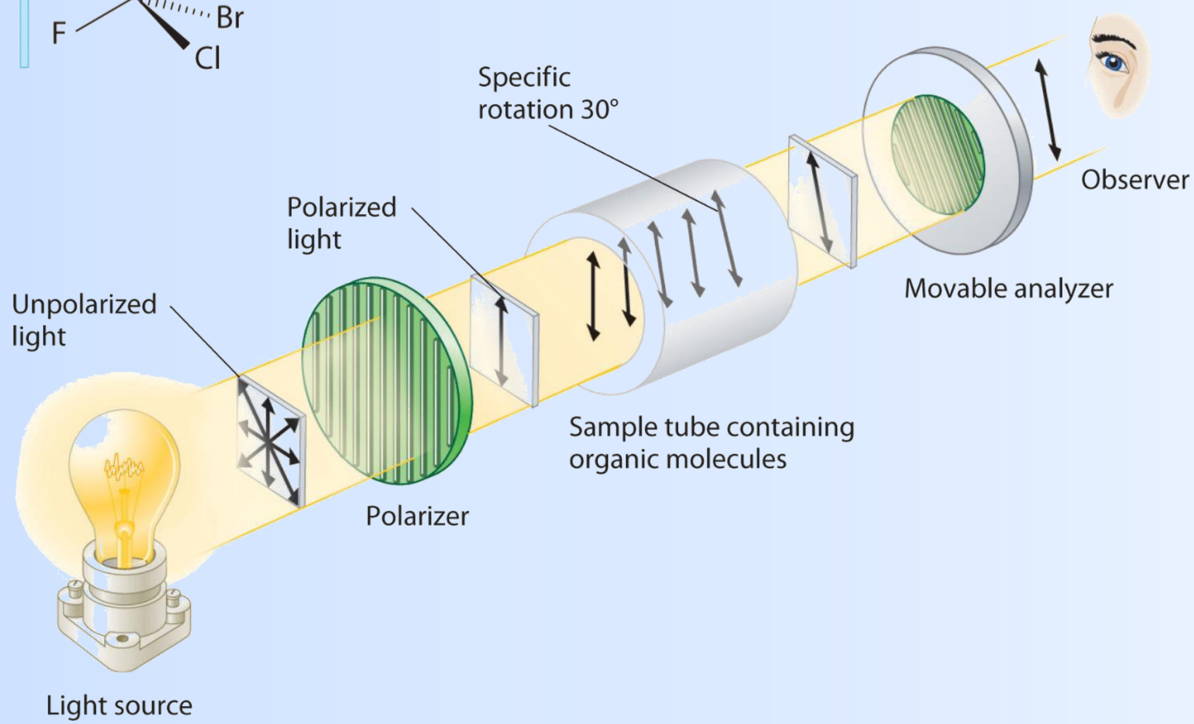


Chiral Materials and Optical Activity



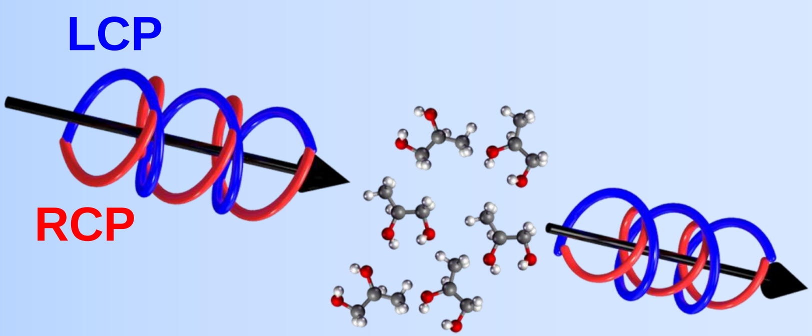
$$\mathbf{D} = \varepsilon \mathbf{E} + i\chi \mathbf{H}$$

$$\mathbf{B} = \mu \mathbf{H} - i\chi \mathbf{E}$$

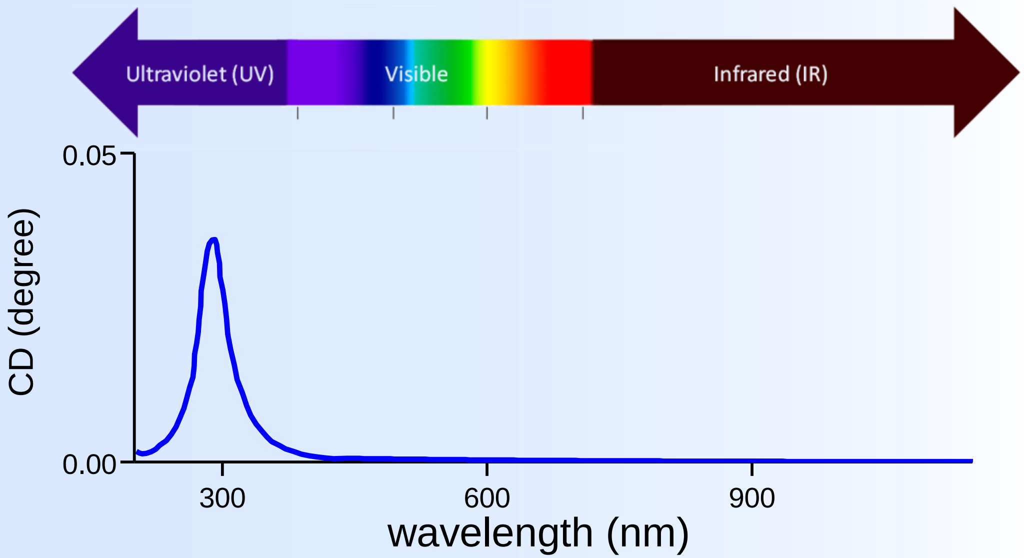


B. A. Averill, *General Chemistry: Principles and Applications* (2007)

Circular Dichroism of Chiral Molecules



$$CD \propto A_{LCP} - A_{RCP}$$



THE FIRST METAMATERIAL ?

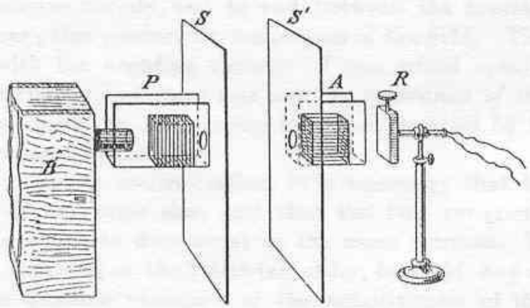


Fig. 1.—Polarisation apparatus. B, the radiating box; P, the polariser; A, the analyser; S, S′, the screens; R, the receiver.

* 'Roy. Proc. Soc.,' vol. 60.

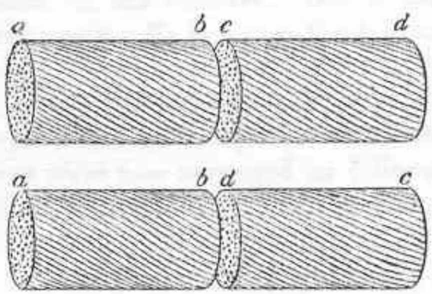
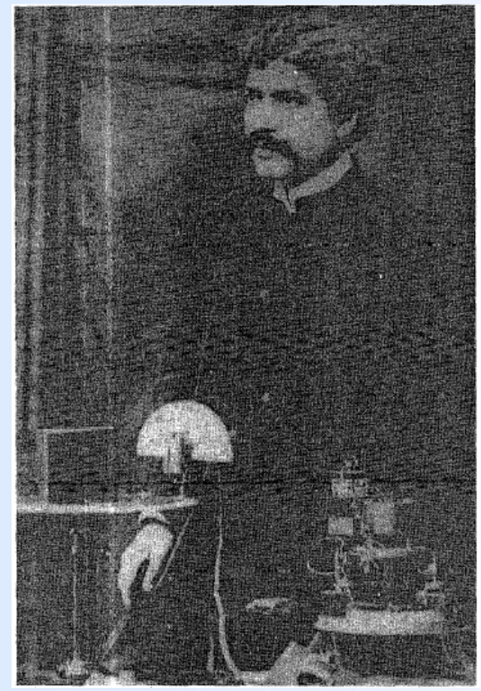
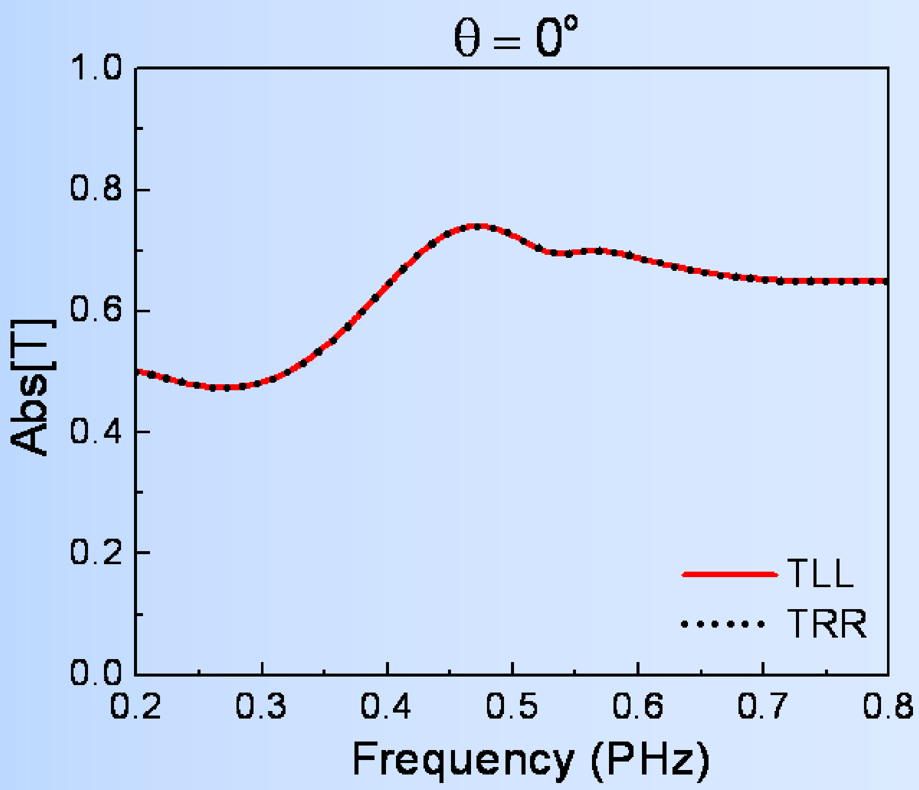


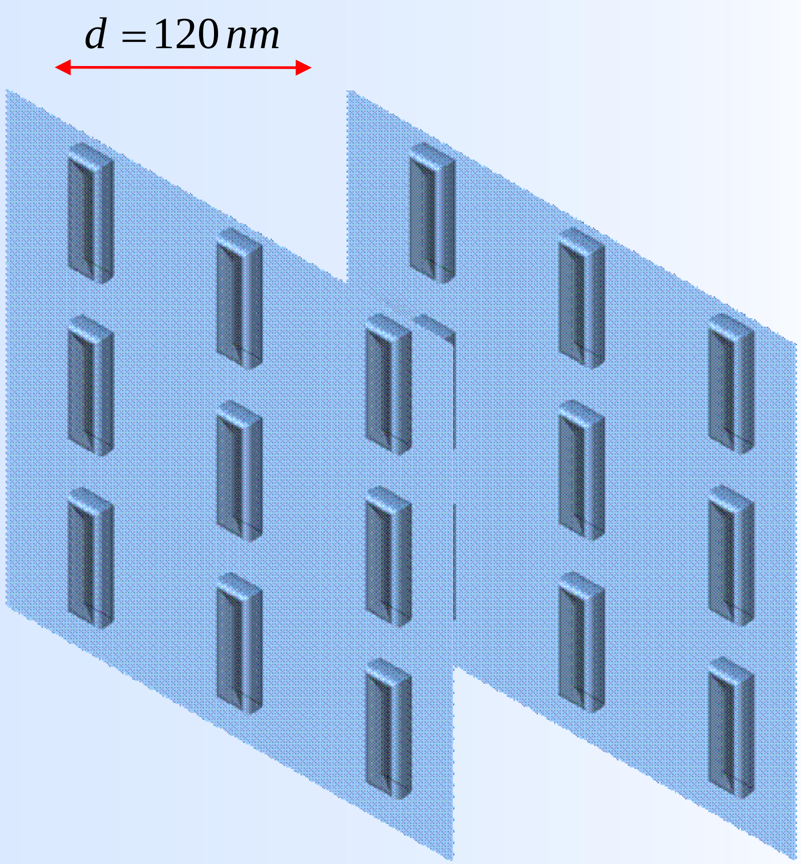
Fig. 2.—Jute elements.



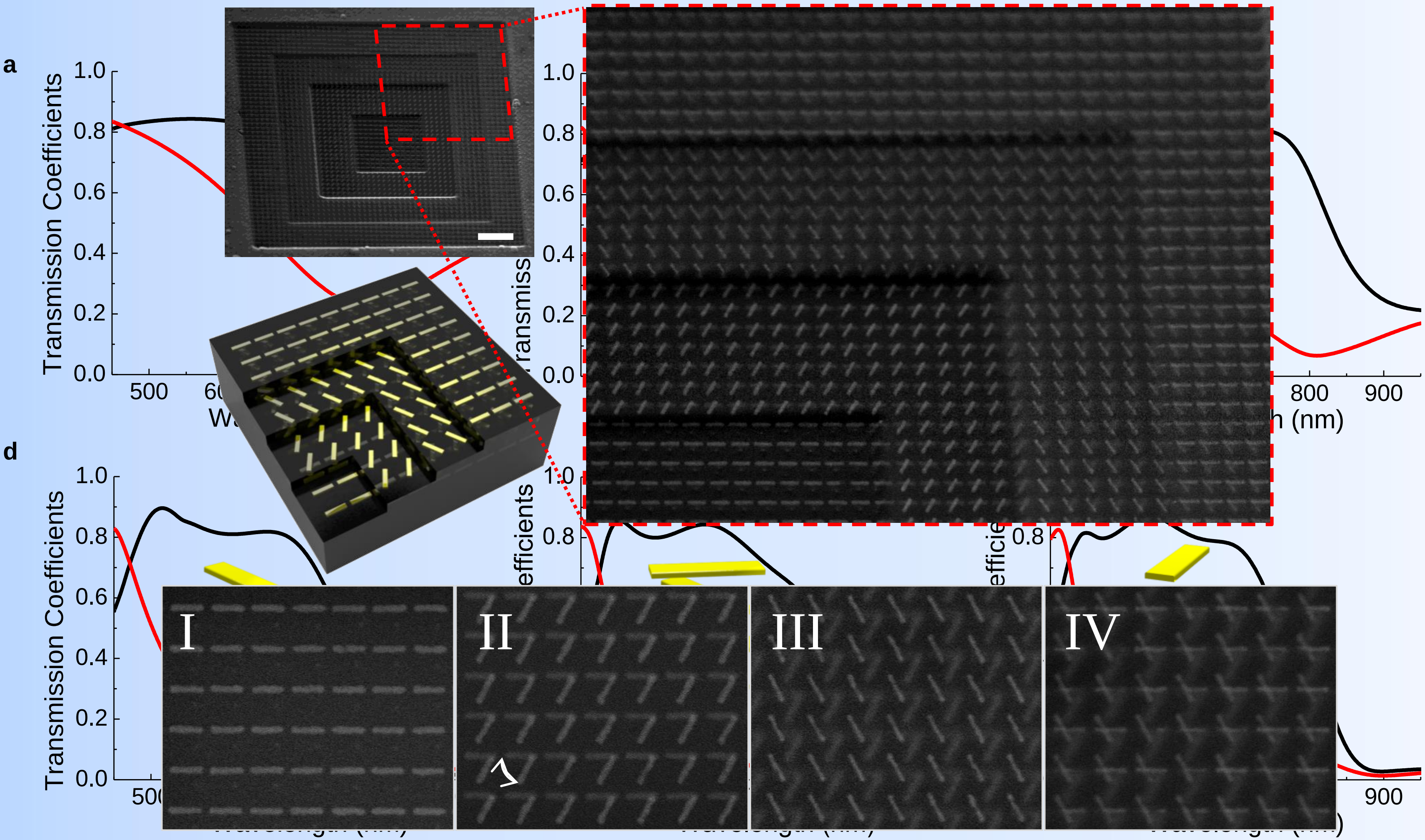
J. C. Bose, *Proc. Royal Society* **63**, 146 (1898)

A Pair of 'Twisted' Metasurfaces



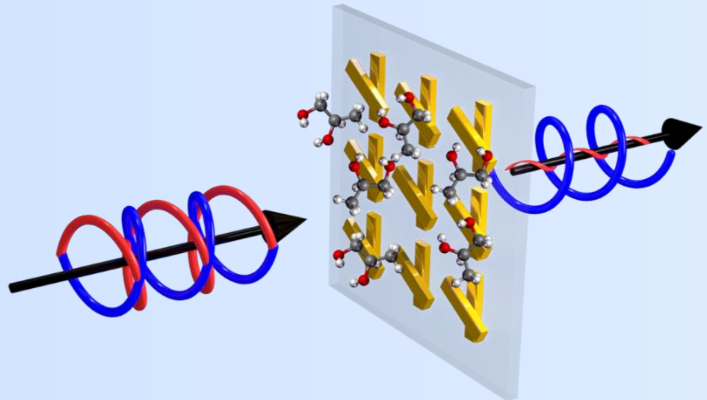


Twisted Metamaterials

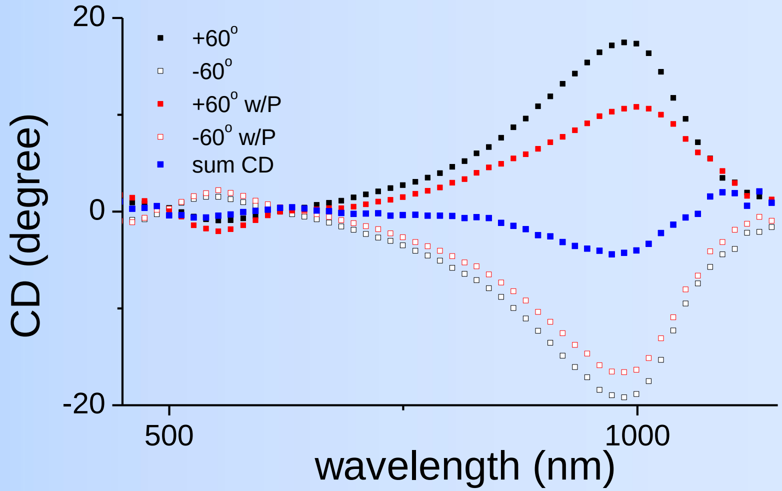


Y. Zhao, M. A. Belkin, A. Alù, *Nature Comm.* **3**, 870 (2012)
Y. Zhao, J. Shi, L. Sun, X. Li, A. Alù, *Adv. Mat.* **26**, 1439 (2014)
Y. Zhao, A. N. Askarpour, L. Sun, J. Shi, X. Li, A. Alù, *Nature Comm.* **8**, 14180 (2017)

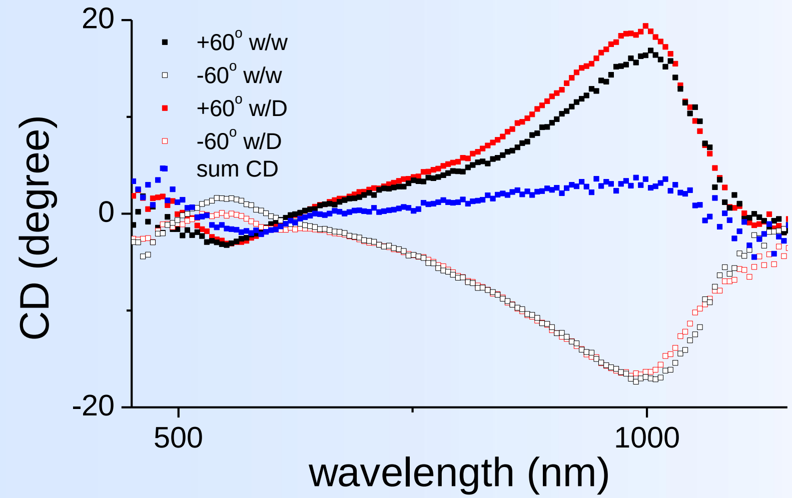
Enhanced Chirality Detection with Metamaterials



Chiral protein: Concanavalin A

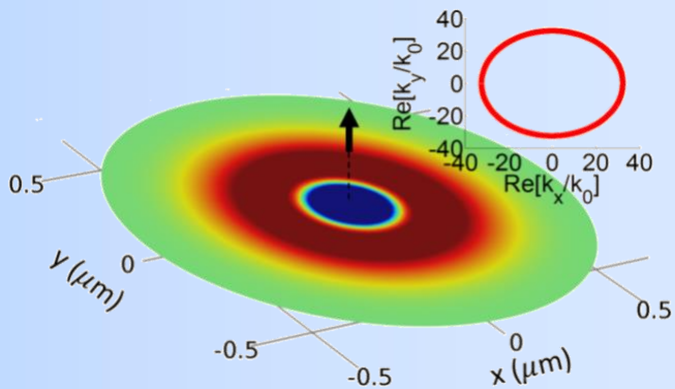


Chiral drug: Irinotecan Hydrochloride



Y. Zhao, A. N. Askarpour, L. Sun, J. Shi, X. Li, A. Alù, *Nature Comm.* **8**, 14180 (2017)

Extreme Anisotropy: Hyperbolic Metasurfaces

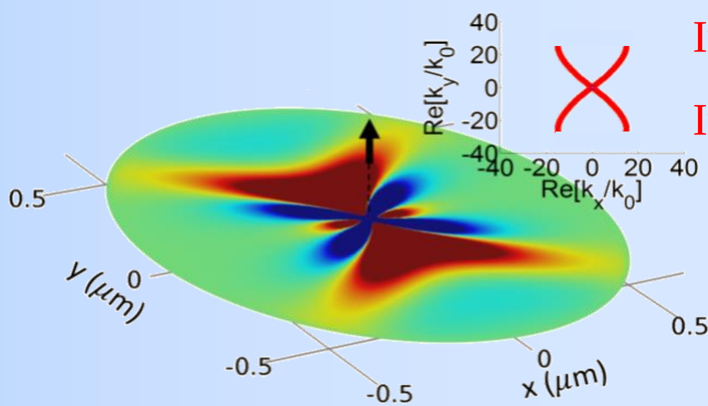


$$\mathrm{Im}[\sigma_{xx}] > 0$$
$$\mathrm{Im}[\sigma_{yy}] > 0$$

Elliptic propagation

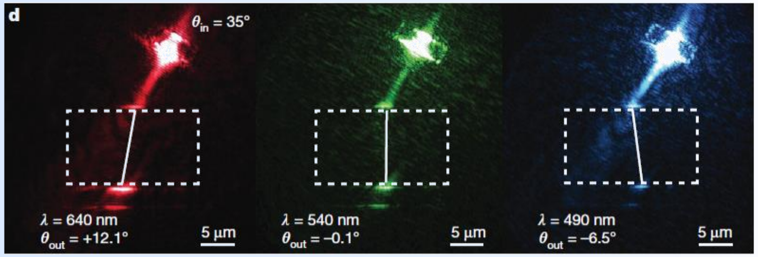
$$\mathbf{J}_{av} = \underline{\boldsymbol{\sigma}} \cdot \mathbf{E}_{av}$$

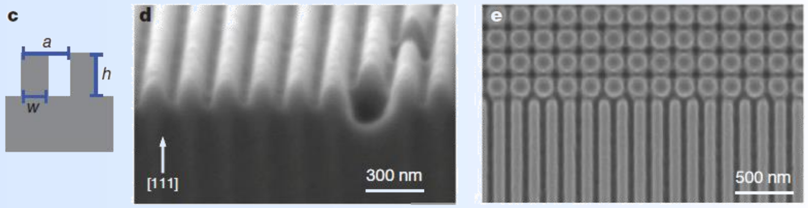
$$\underline{\boldsymbol{\sigma}} = \begin{pmatrix} \sigma_{xx} & 0 \\ 0 & \sigma_{yy} \end{pmatrix}$$



$$\mathrm{Im}[\sigma_{xx}] > 0$$
$$\mathrm{Im}[\sigma_{yy}] < 0$$

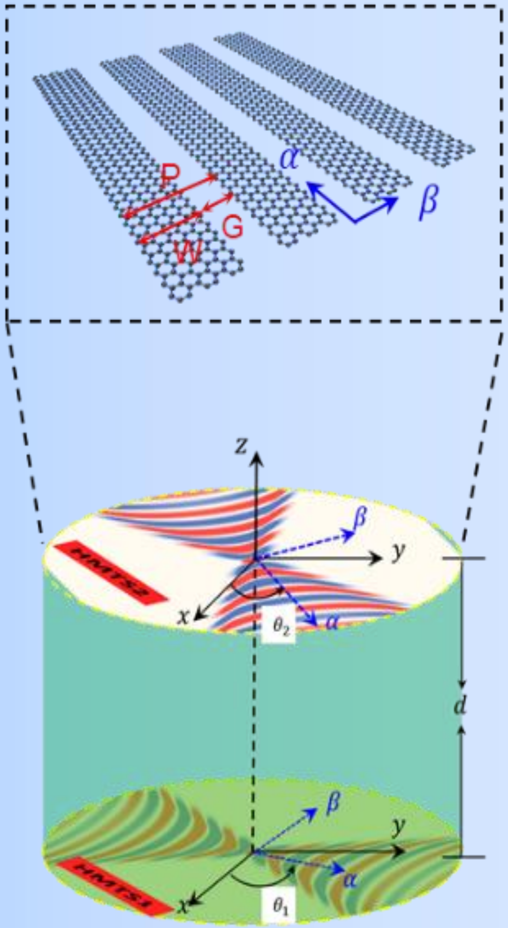
Hyperbolic propagation

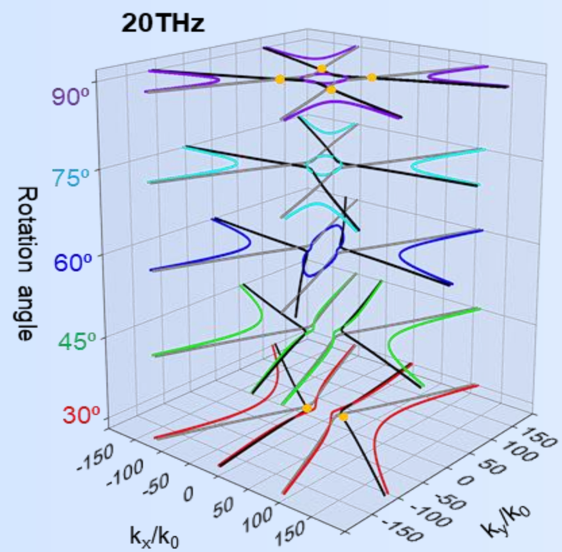


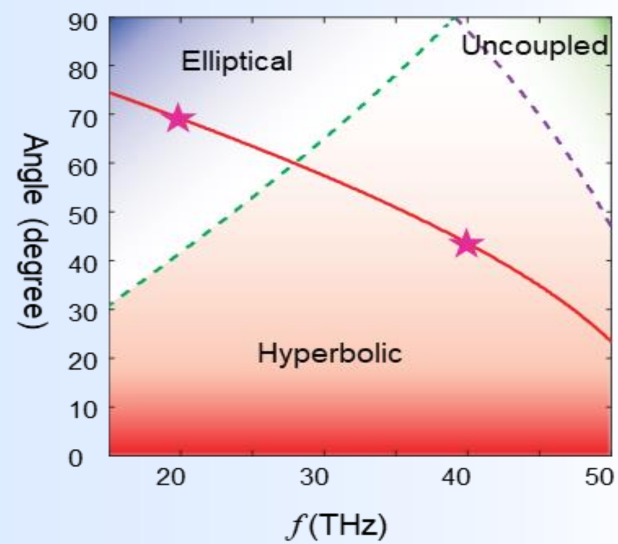


J. S. Gomez-Diaz, et al., *Phys. Rev. Lett.* **114**, 233901 (2015)
A. A. High, et al. *Nature* **522**, 192 (2015)

Twisted Hyperbolic Metasurfaces



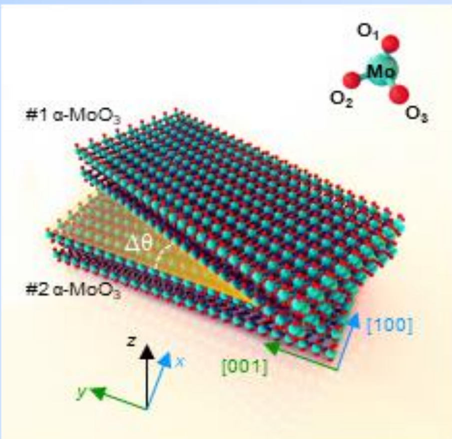


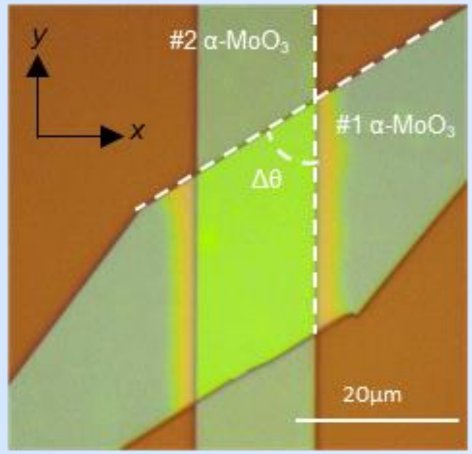


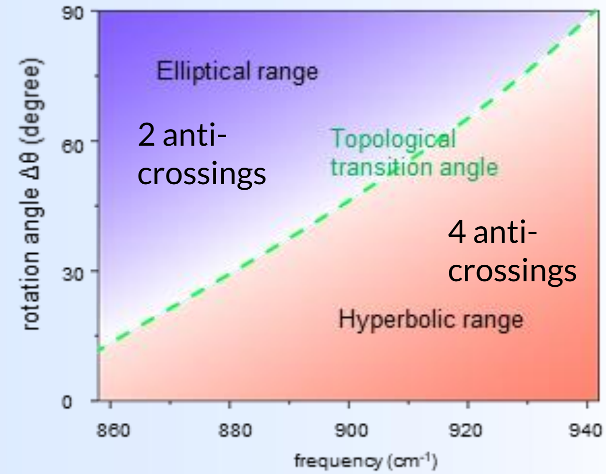
G. Hu, A. Krasnok, Y. Mazor, C. W. Qiu, A. Alù, *Nano Letters* **20**, 3217 (2020)

Twisted α-MoO_3 Bilayers

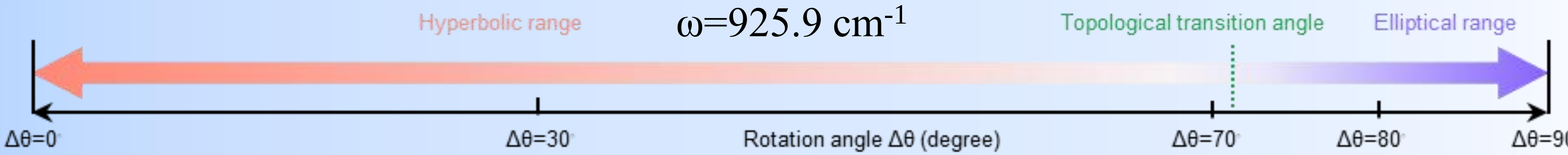
$\Delta\theta = 57^\circ$, $d_1 = d_2 = 150$ nm







$\omega = 925.9$ cm^{-1}

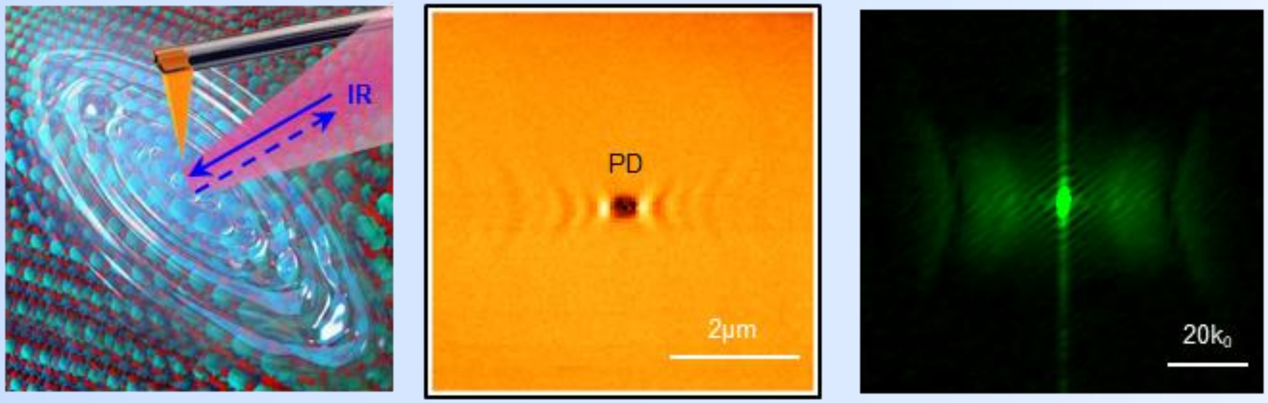


G. Hu, et al., *Nano Letters* **20**, 3217 (2020)
G. Hu, et al., *Nature* **582**, 209 (2020)

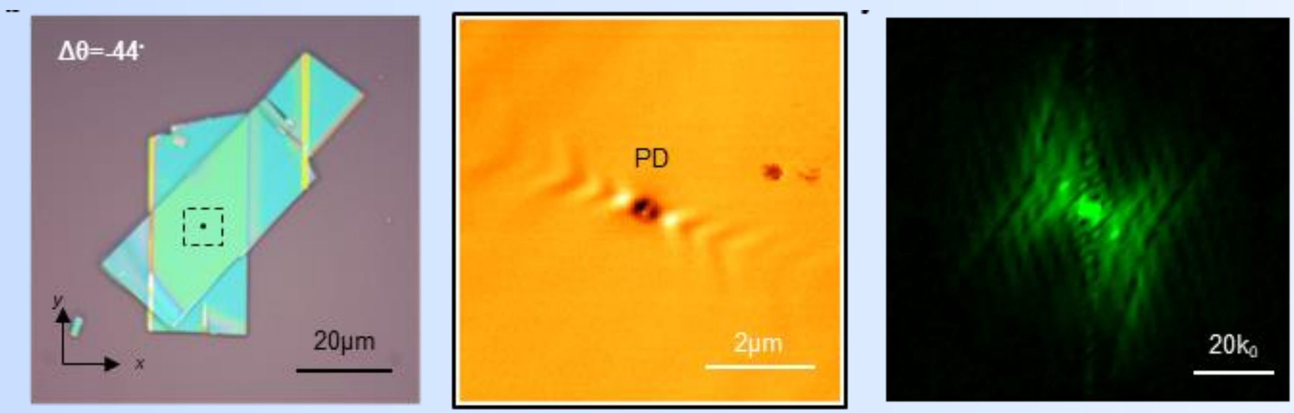
EXPERIMENTAL VERIFICATION IN TWISTED α-MoO_3 BILAYERS

ω=903.8 cm^{-1}

Single layer



Bi-layer Δθ=-44°

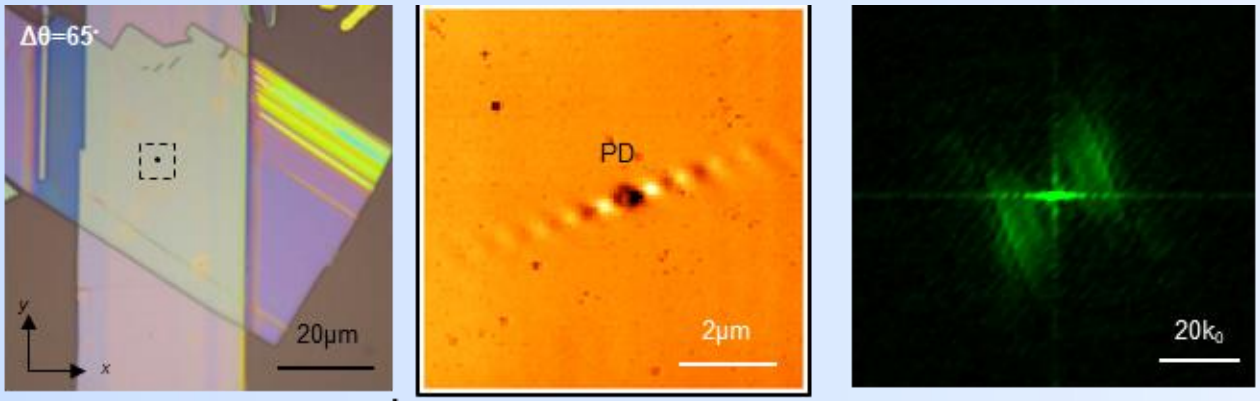


G. Hu, et al., *Nature* **582**, 209 (2020)

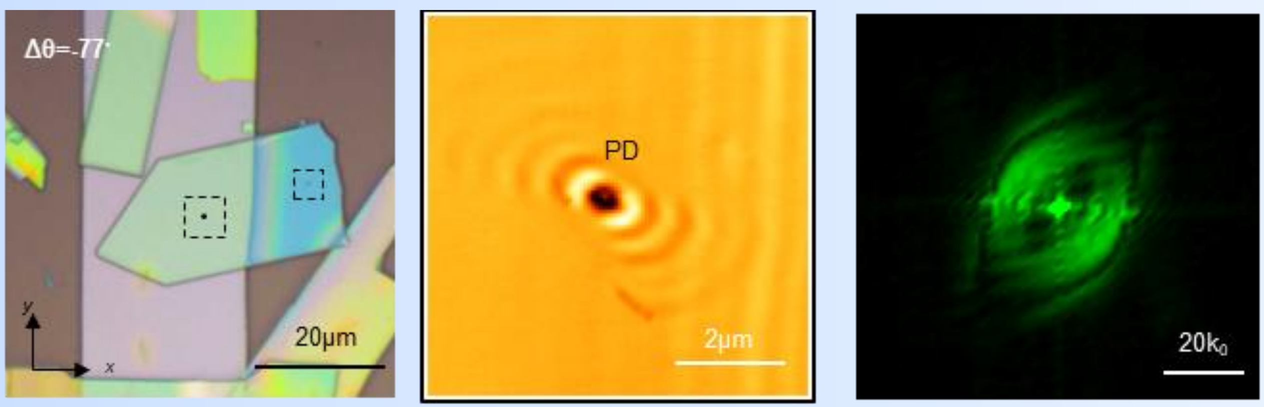
Experimental Verification in Twisted α-MoO_3 Bilayers

ω=903.8 cm^{-1}

Bi-layer Δθ=65°

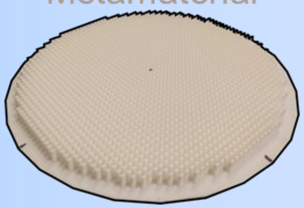


Bi-layer Δθ=-77°

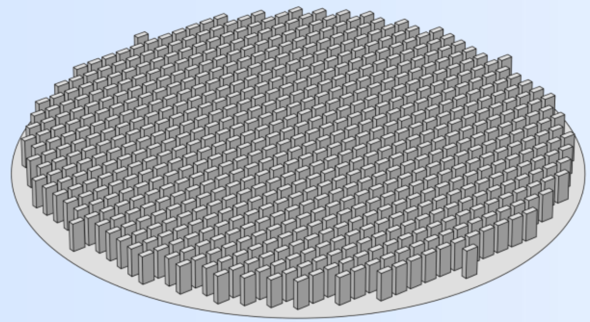


G. Hu, et al., *Nature* **582**, 209 (2020)

Hyperbolic Waves in Elastic Metasurfaces

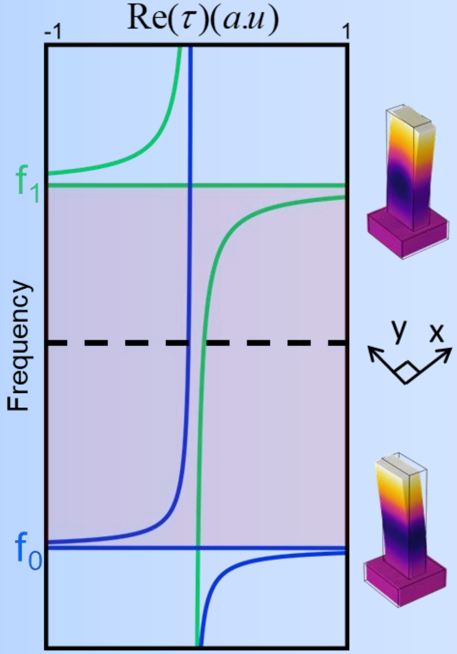


$$\underline{\underline{\tau}}_{xy} = \begin{pmatrix} \tau_0 < 0 & 0 \\ 0 & \tau_1 > 0 \end{pmatrix}$$

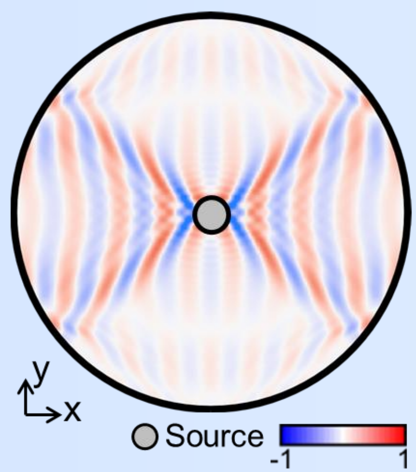


Dispersion relation

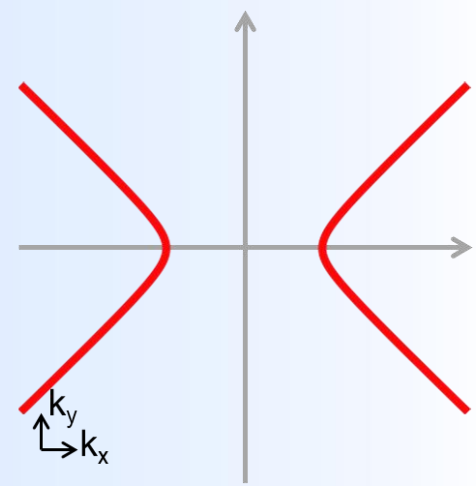
$$\tau_0 k_x^{\ 2} + \tau_1 k_y^{\ 2} = K\omega^2$$



Orthogonal resonances



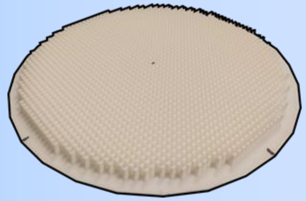
Hyperbolic wavefronts



Open
iso-Frequency Contour

Hyperbolic Waves in Elastic Metasurfaces

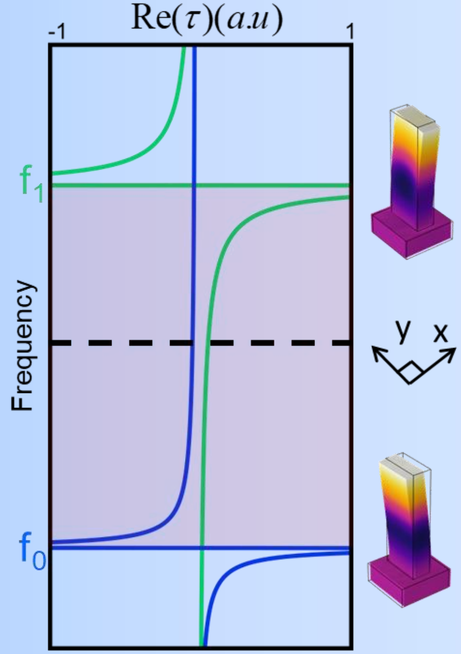
Metamaterial



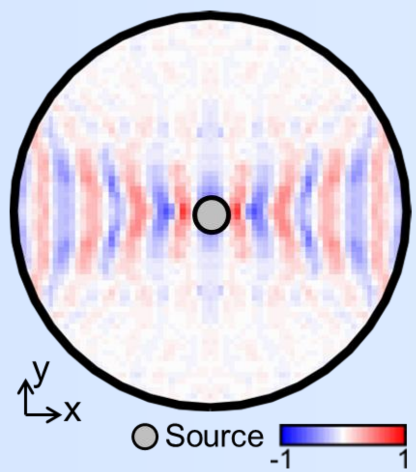
$$\underline{\underline{\tau}}_{xy} = \begin{pmatrix} \tau_0 < 0 & 0 \\ 0 & \tau_1 > 0 \end{pmatrix}$$

Dispersion relation

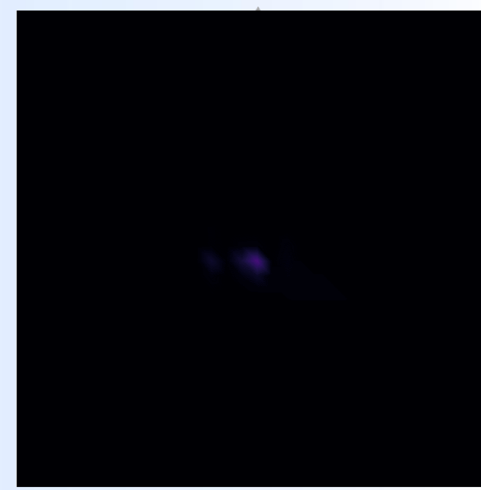
$$\tau_0 k_x^{\,2} + \tau_1 k_y^{\,2} = K\omega^2$$



Orthogonal resonances

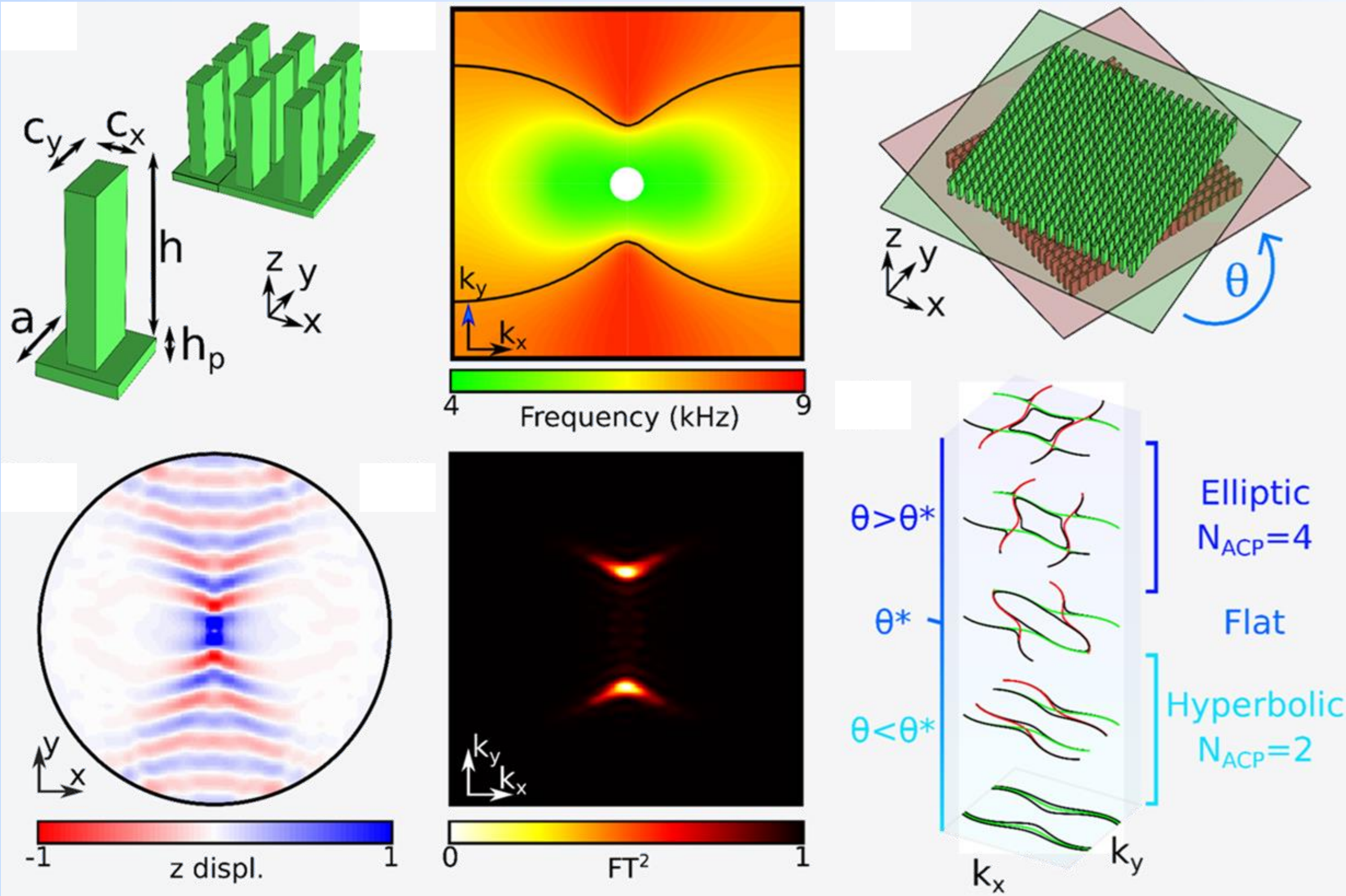


Hyperbolic wavefronts

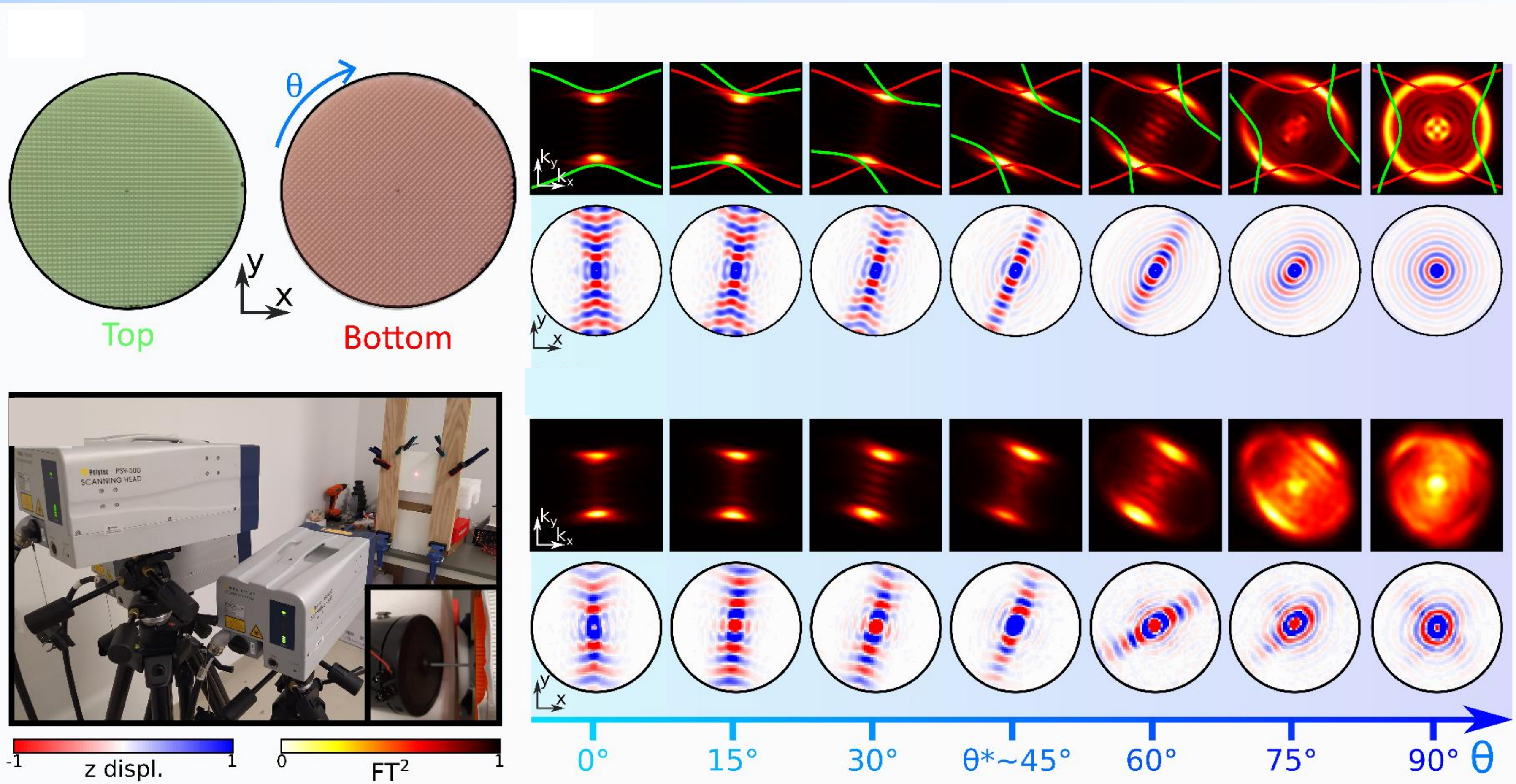


Open
iso-Frequency Contour

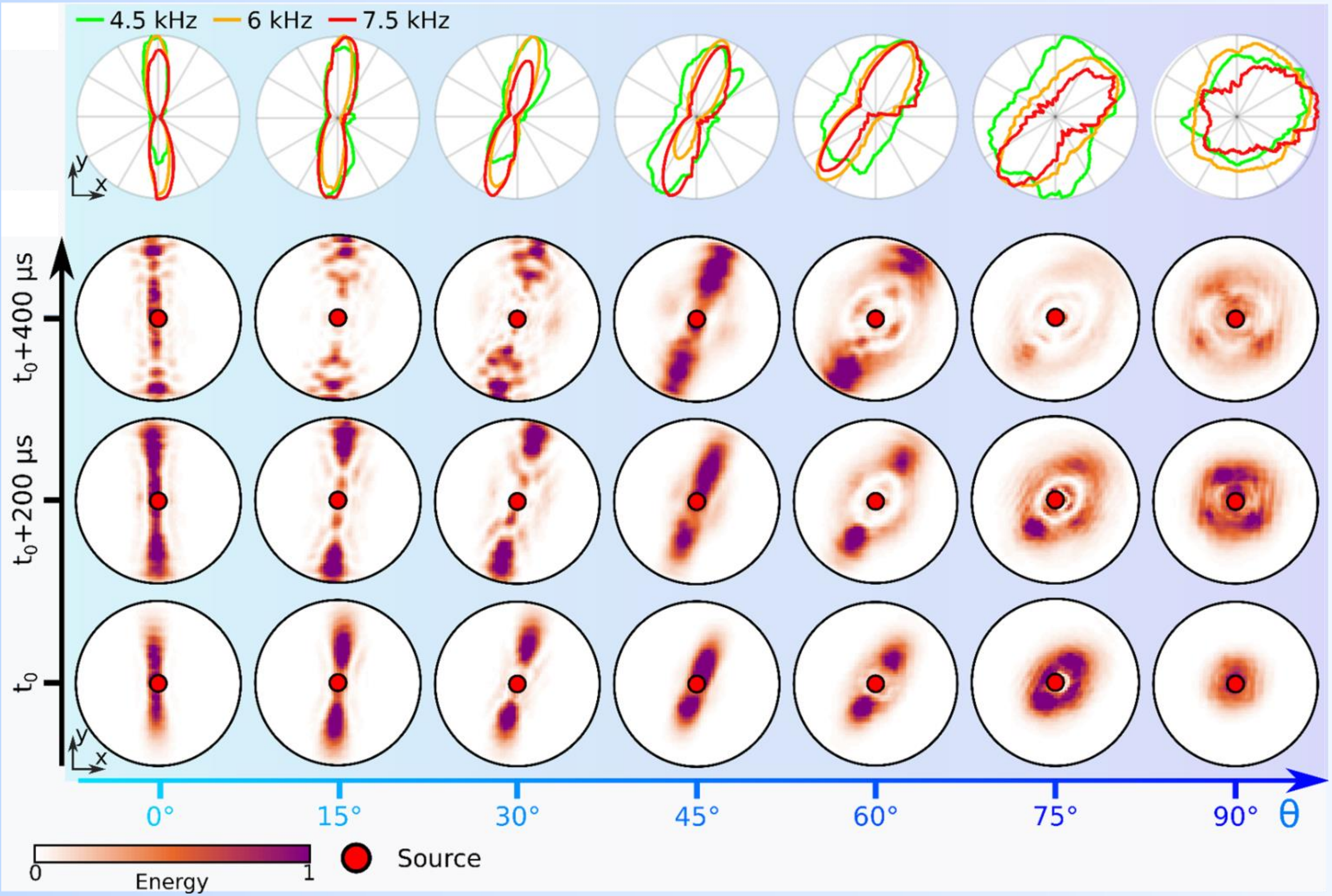
Twisted Hyperbolic Metasurfaces for Elastic Waves



Twisted Hyperbolic Metasurfaces for Elastic Waves

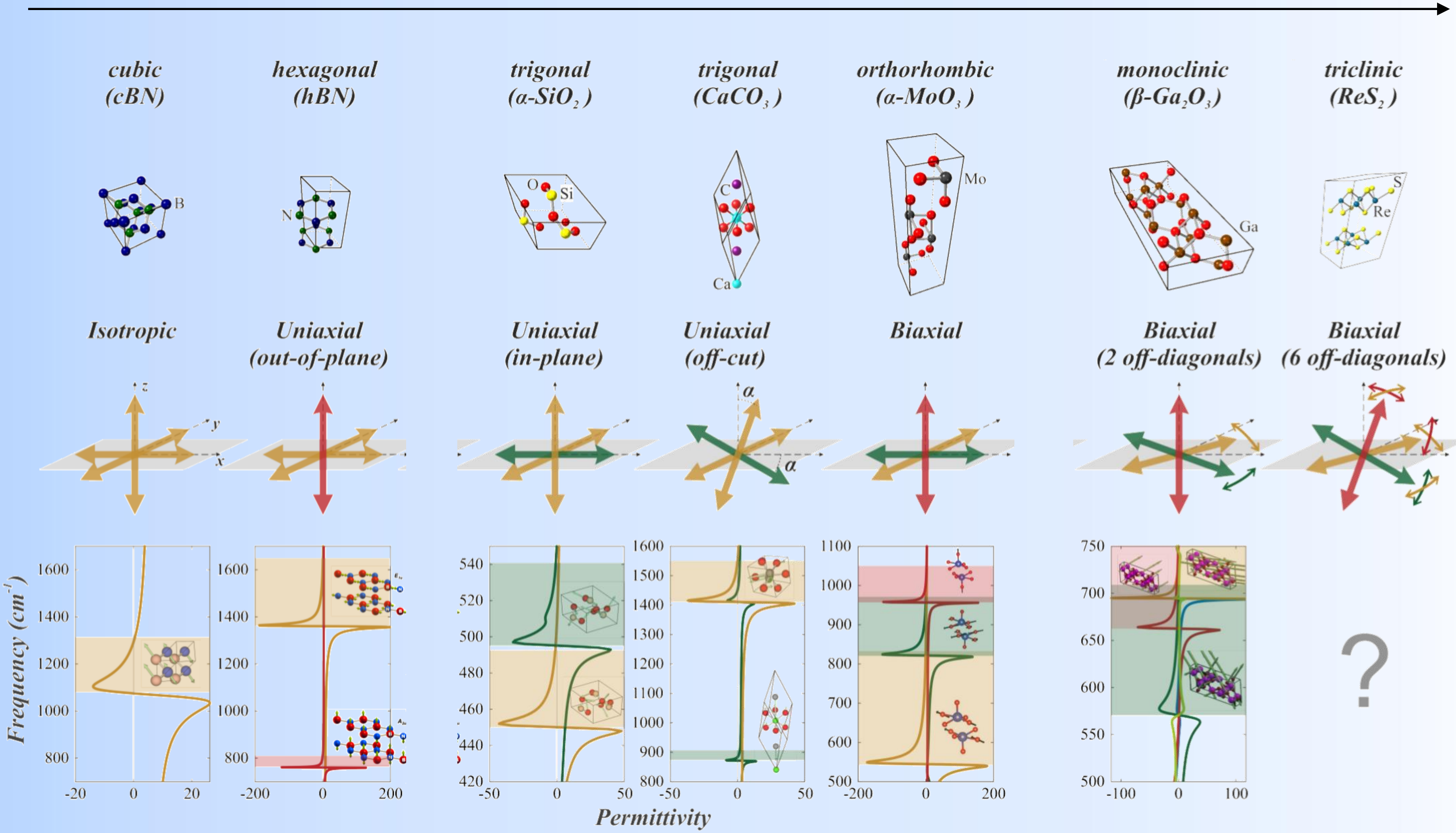


Twisted Hyperbolic Metasurfaces for Elastic Waves



Phonon Polariton Crystals

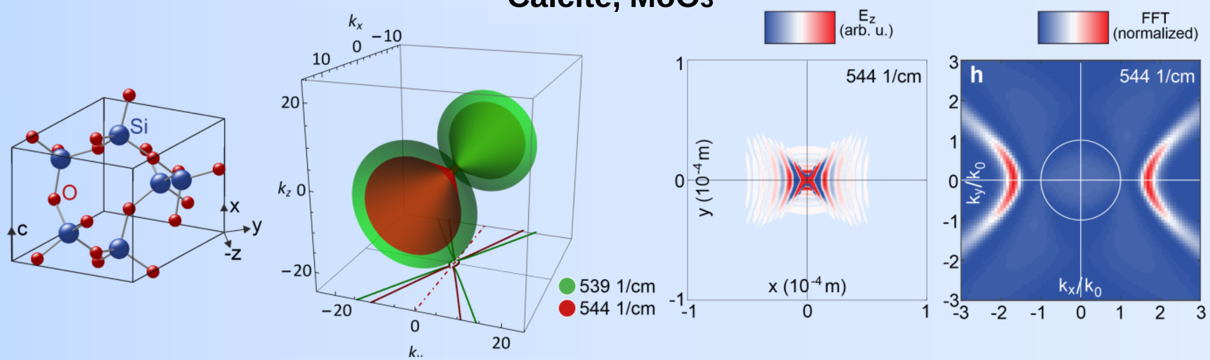
Broken crystal symmetry in nature



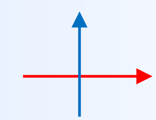
E. Galiffi, G. Carini, et al., *Nature Rev. Materials* **9**, 9 (2024)

Monoclinic Crystals

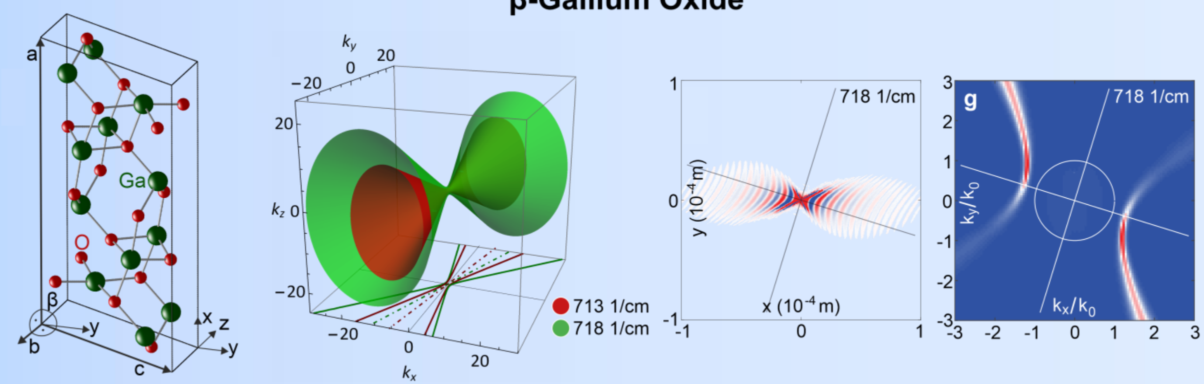
Calcite, MoO_3



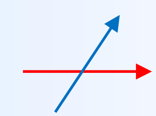
$$\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\parallel & 0 \\ 0 & 0 & \varepsilon_\perp \end{bmatrix}$$



β-Gallium Oxide

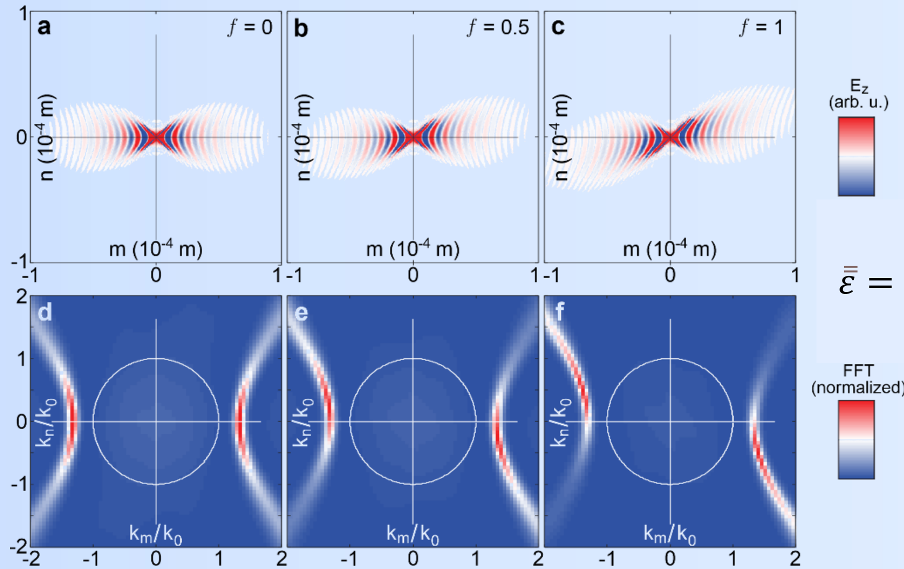


$$\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{xy} & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}$$

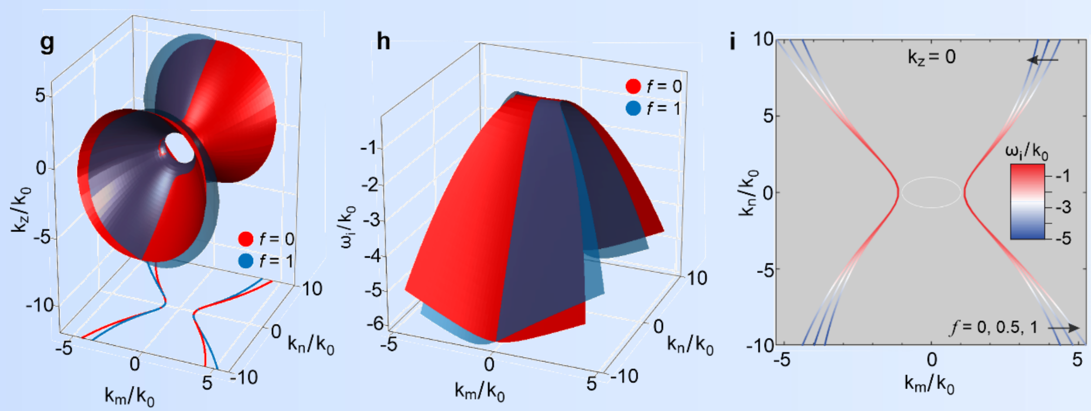


N. C. Passler, et al., *Nature* **602**, 599 (2022)

Role of Non-Hermiticity and Low-Symmetry

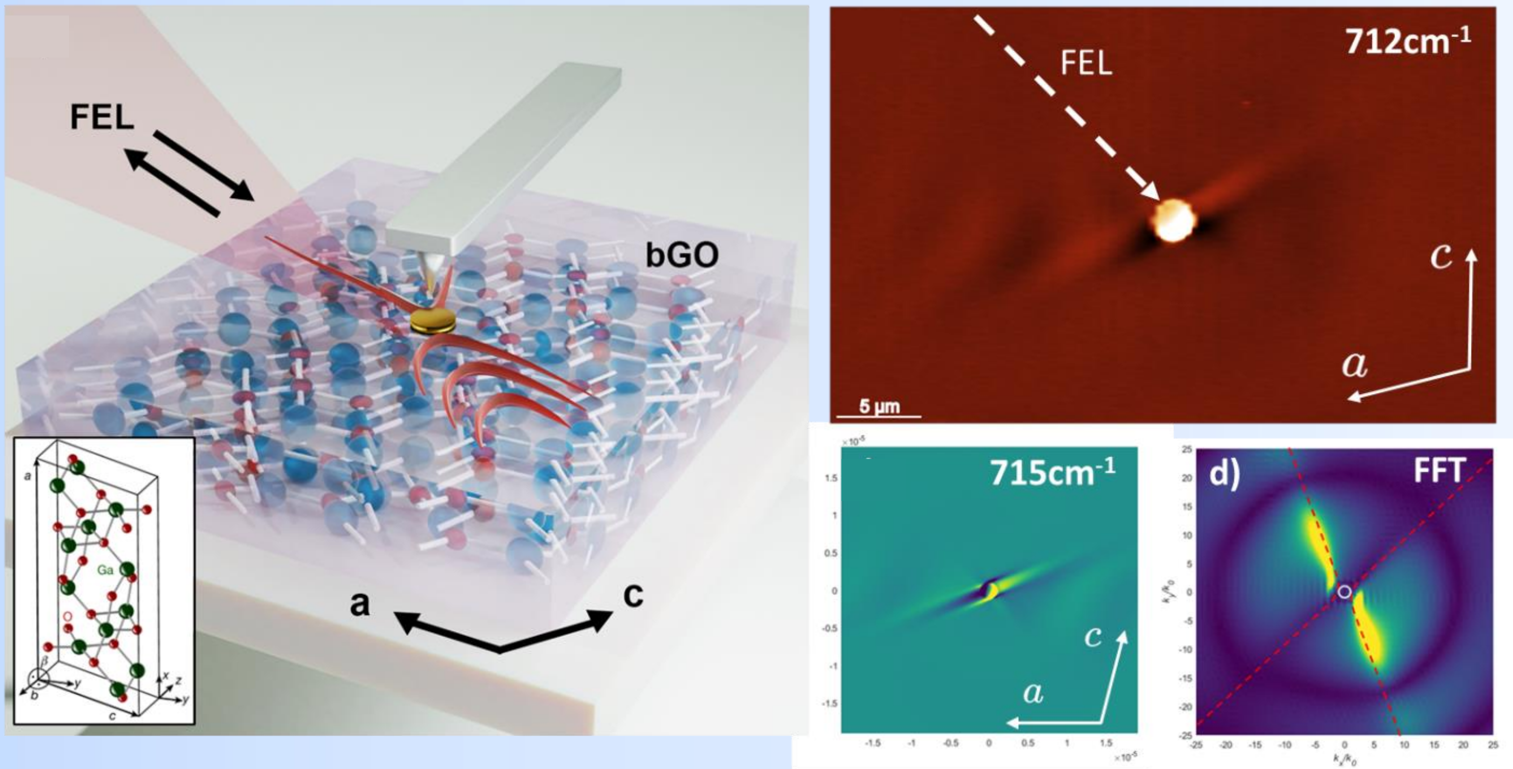


$$\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_{mm} & i\,f\,\varepsilon_{bGO} & 0 \\ i\,f\,\varepsilon_{bGO} & \varepsilon_{nn} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}$$



N. Passler, et al., *Nature* **602**, 599 (2022)

Real-Space Observation of Hyperbolic Shear Polaritons



J. Matson, et al., *Nature Communications* **14**, 5240 (2023) [in β-Ga_2O_3]
G. Hu, et al., *Nature Nanotechnology* **18**, 64 (2023) [in $CdWO_4$]

Hyperbolic Shear Metasurfaces

$$\underline{\boldsymbol{\sigma}} = \begin{pmatrix} \sigma_{xx} & 0 \\ 0 & \sigma_{yy} \end{pmatrix}$$

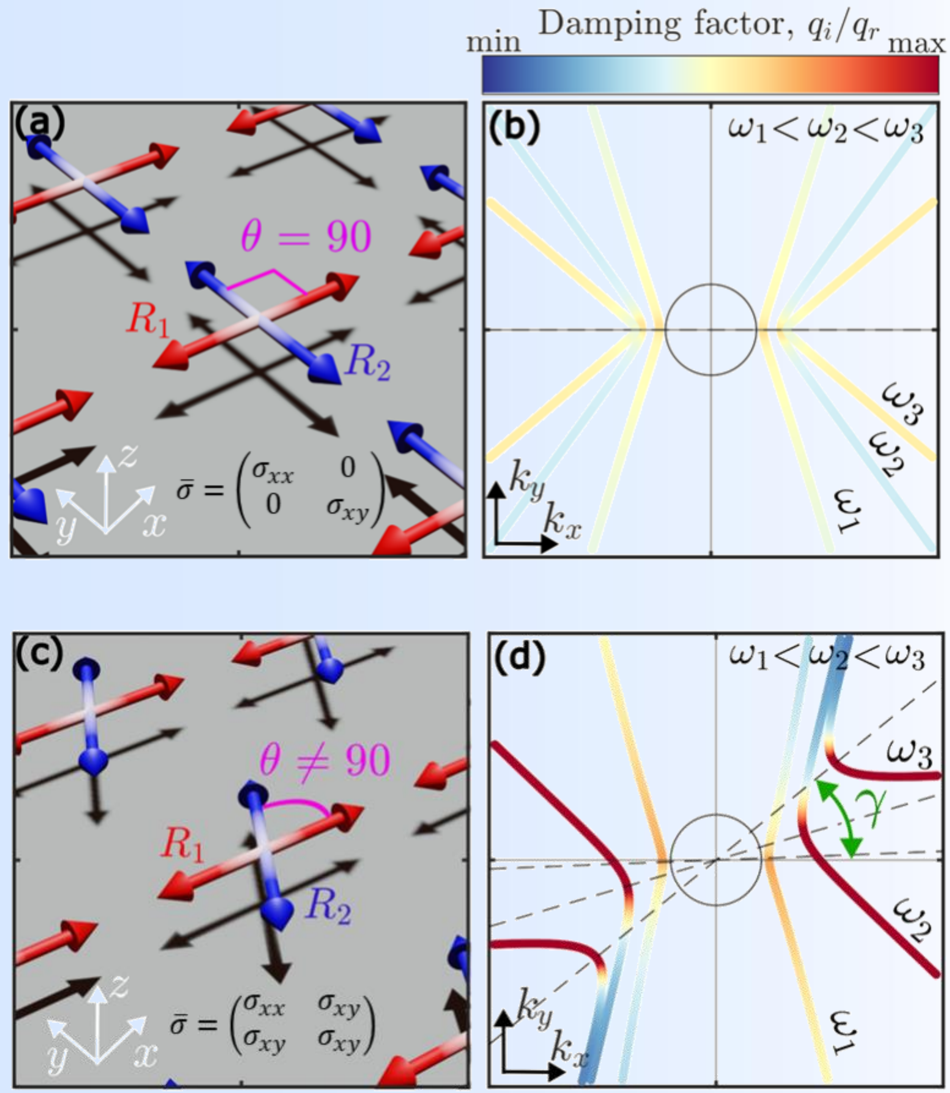
Surface conductivity:

$$\bar{\sigma} = \begin{pmatrix} \sigma_1 & 0 \\ 0 & 0 \end{pmatrix} + \bar{R}_\theta \begin{pmatrix} \sigma_1 & 0 \\ 0 & 0 \end{pmatrix} \bar{R}_\theta^T$$

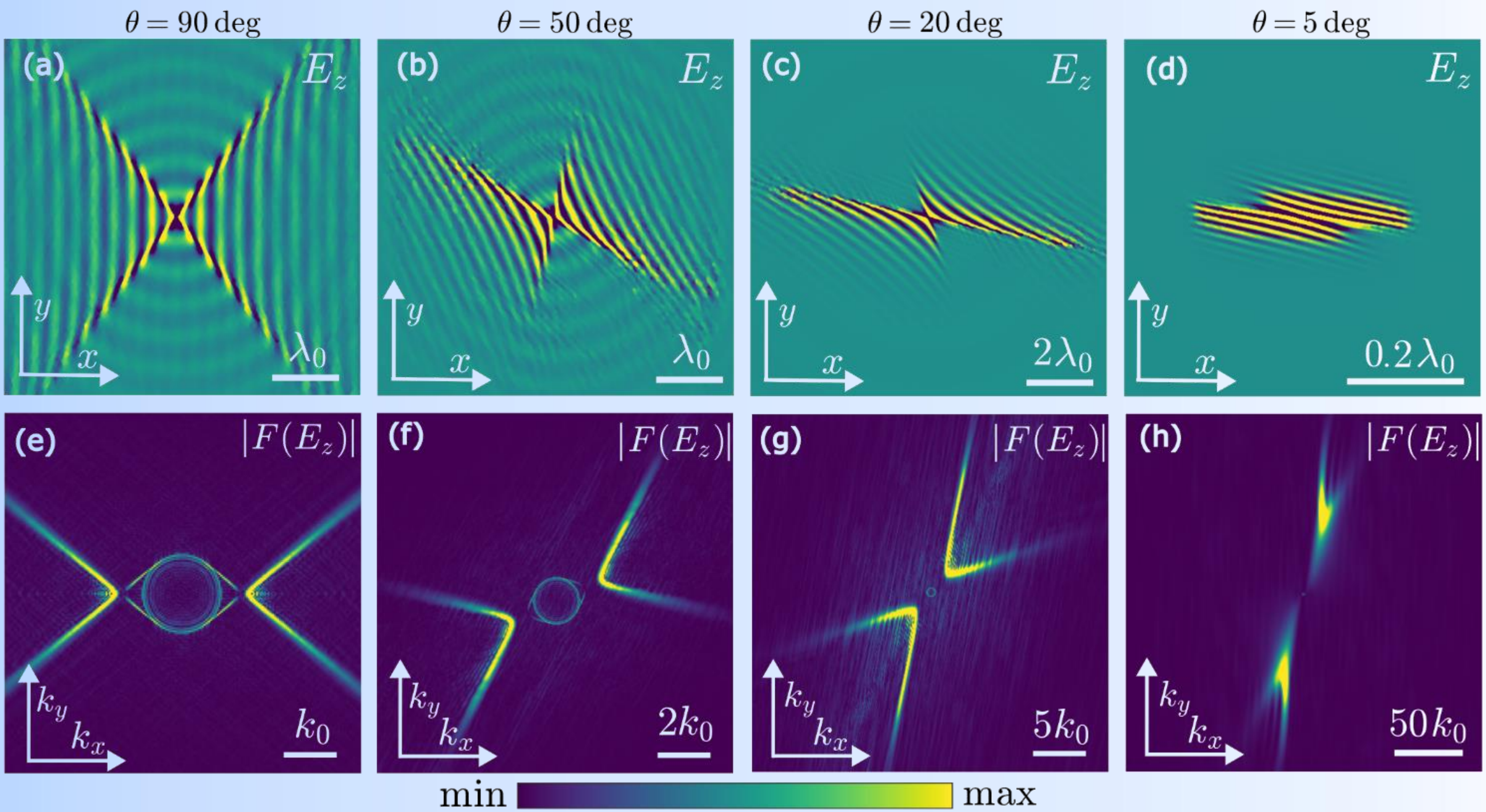
where:

$$\sigma_i \propto \frac{\omega^2}{-\omega_0^2 + \omega^2 + i\Gamma\omega}$$

$$\bar{R}_\theta = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

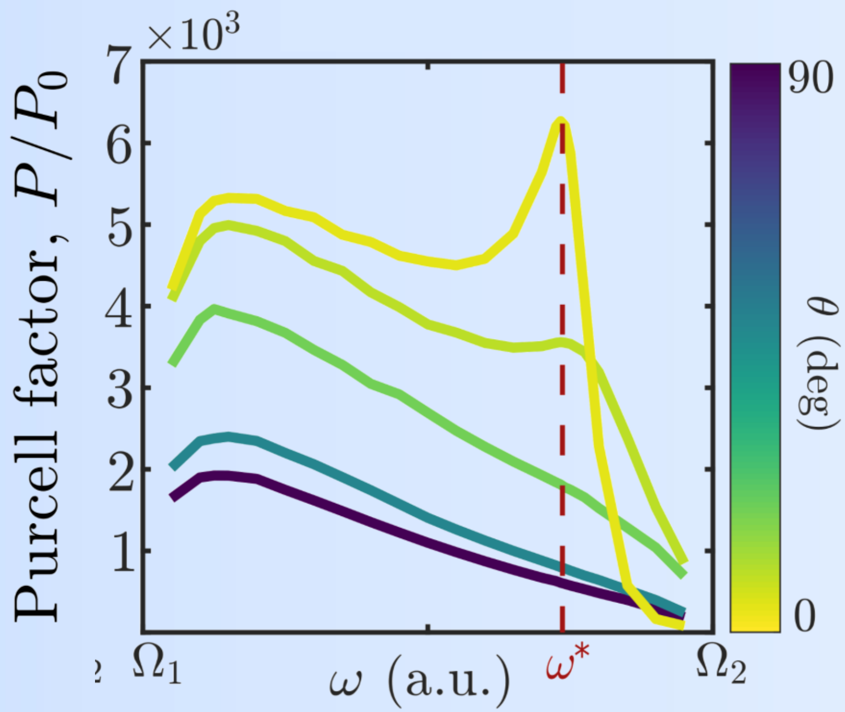


Shear Hyperbolic Metasurfaces

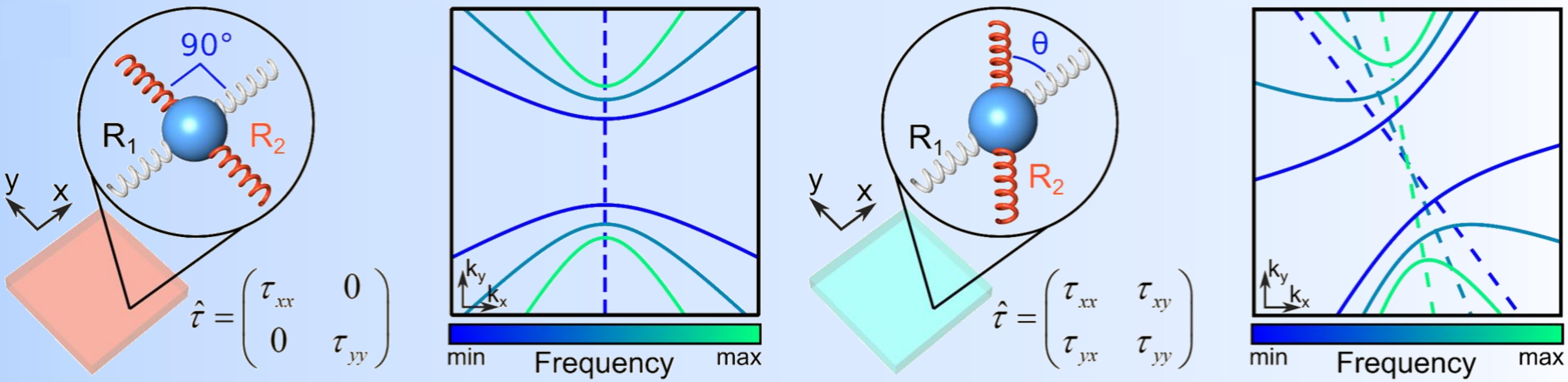


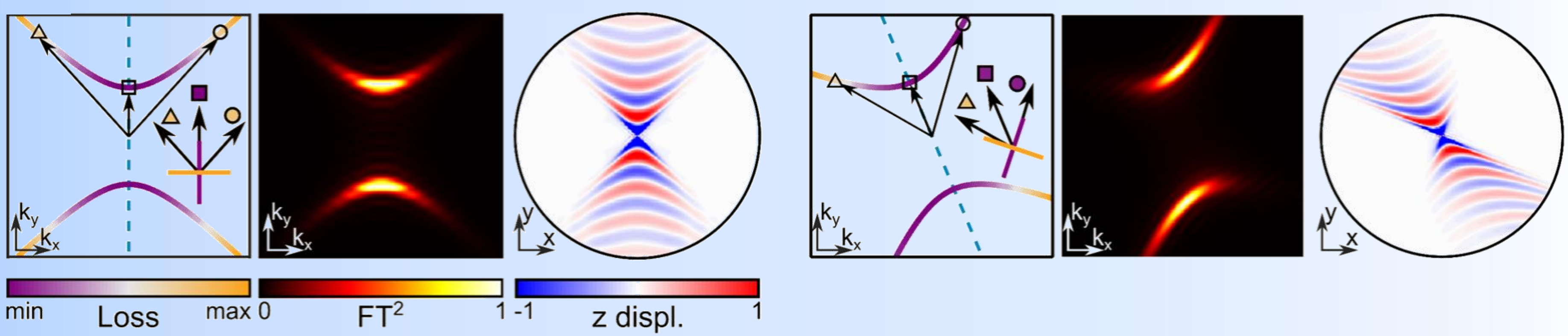
SHEAR HYPERBOLIC METASURFACES

Purcell factor enhanced by loss redistribution driven by broken symmetry

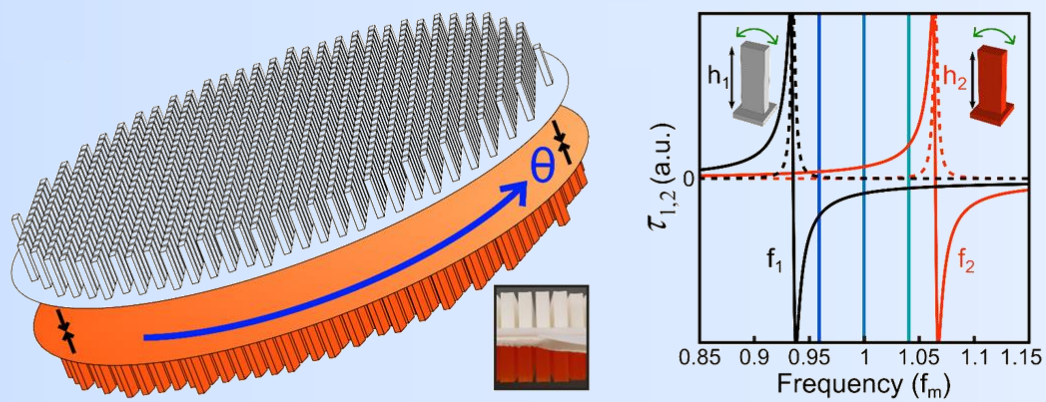


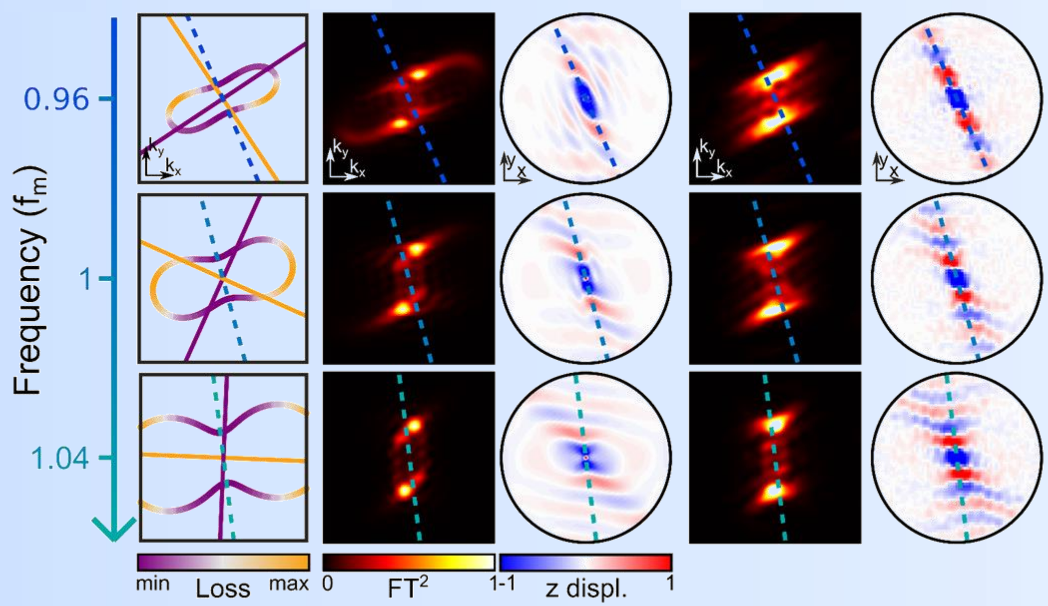
Hyperbolic Shear Waves in Elastic Metasurfaces



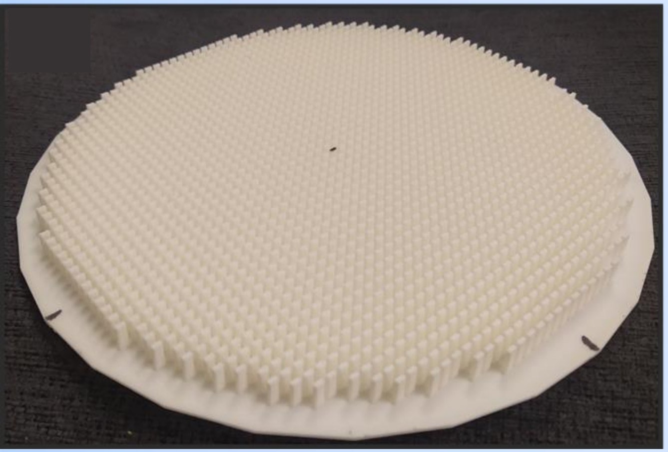


Hyperbolic Shear Waves in Elastic Metasurfaces



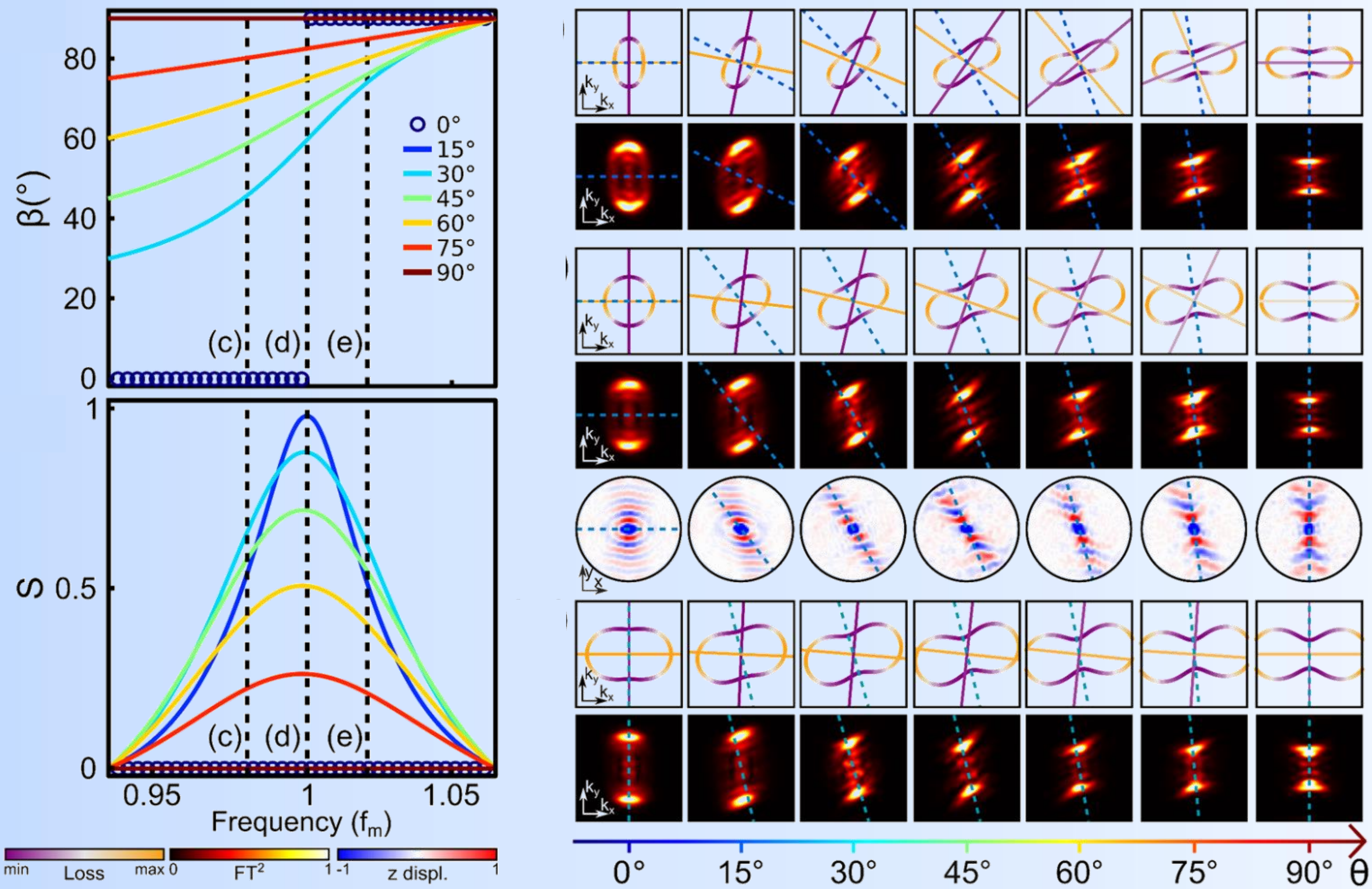


Shear Hyperbolic Waves in Elastic Metasurfaces



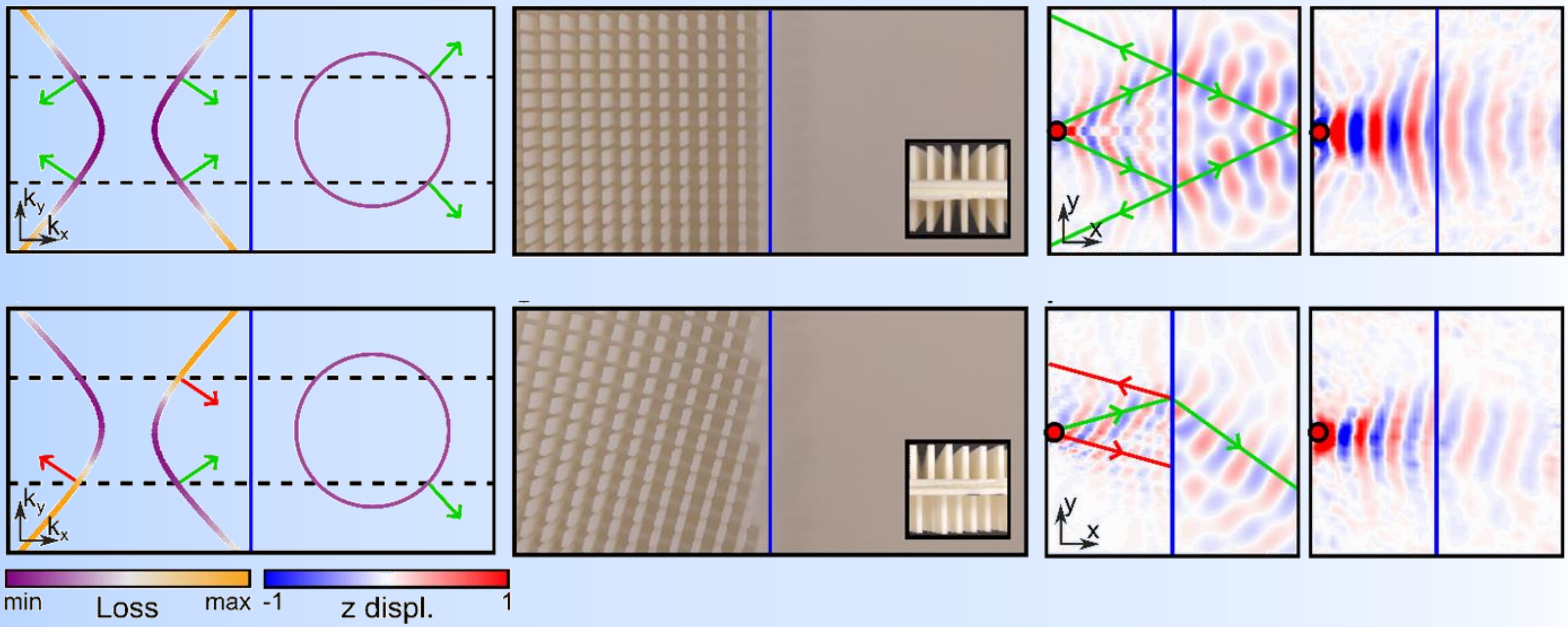
S. Yves, E. Galiffi, X. Ni, E. M. Renzi, A. Alù, *Physical Review X*, in press (2024)

Shear Hyperbolic Waves in Elastic Metasurfaces

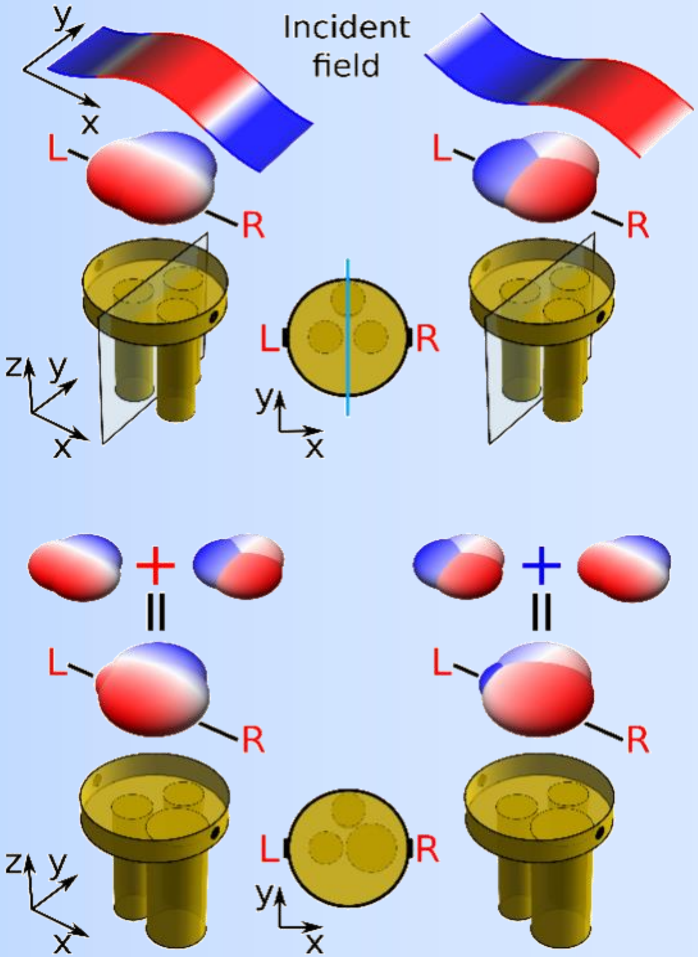


S. Yves, E. Galiffi, X. Ni, E. M. Renzi, A. Alù, *Physical Review X*, in press (2024)

Reflection-Less Negative Refraction



Chirality in Acoustics

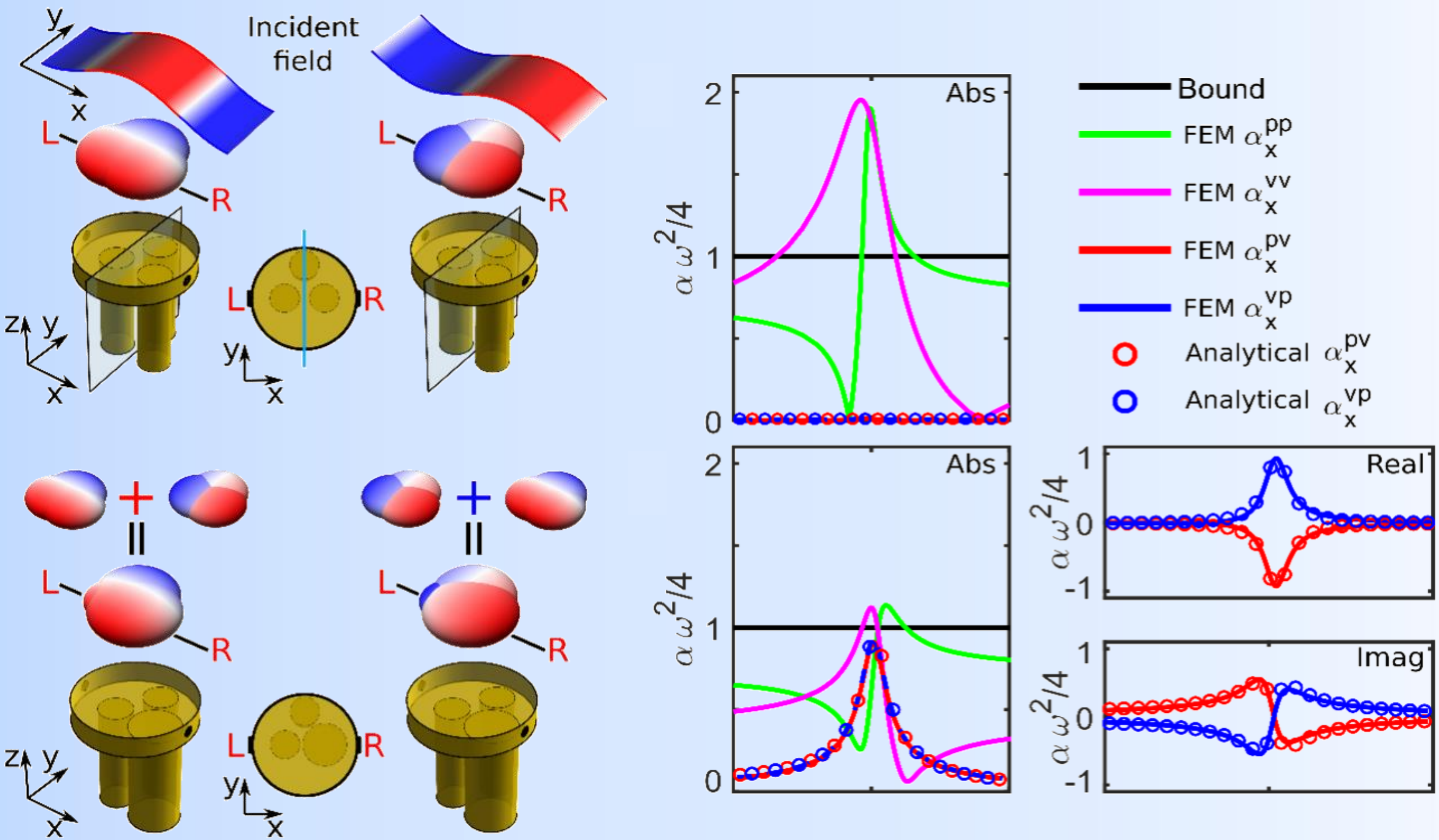


$$\begin{pmatrix} M \\ \mathbf{D} \end{pmatrix} = \begin{pmatrix} \alpha_{pp} & 0 \\ 0 & \boldsymbol{\alpha}_{vv} \end{pmatrix} \begin{pmatrix} p_{loc} \\ \mathbf{v}_{loc} \end{pmatrix}$$

$$\begin{pmatrix} M \\ \mathbf{D} \end{pmatrix} = \begin{pmatrix} \alpha_{pp} & \boldsymbol{\alpha}_{pv} \\ \boldsymbol{\alpha}_{vp} & \boldsymbol{\alpha}_{vv} \end{pmatrix} \begin{pmatrix} p_{loc} \\ \mathbf{v}_{loc} \end{pmatrix}$$

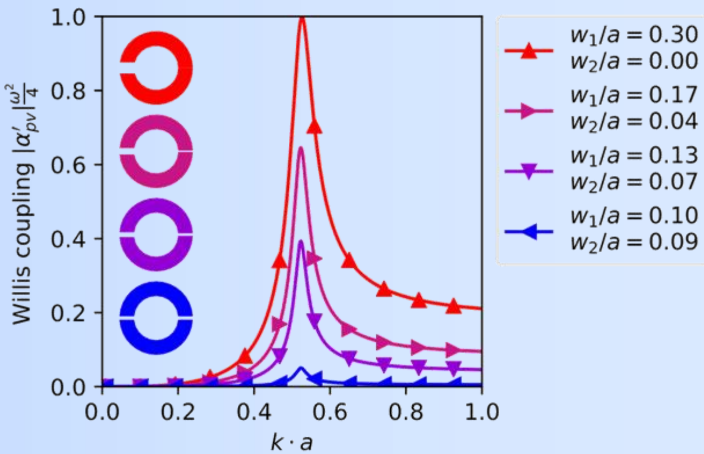
J. R. Willis, *Wave Motion* **3**, 1 (1981)
G. W. Milton, M. Briane, J. R. Willis, *New J. Phys.* **8**, 246 (2006)

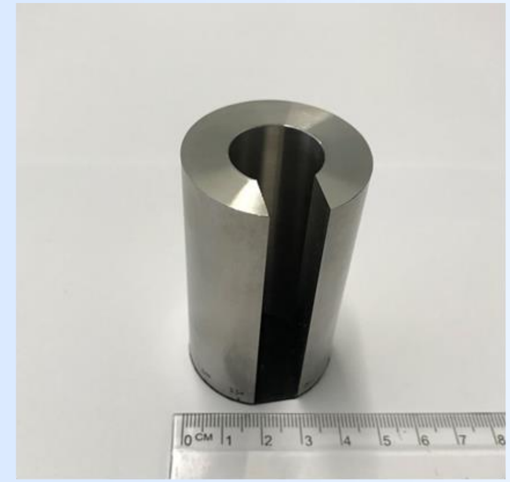
Even-Symmetric Willis Coupling

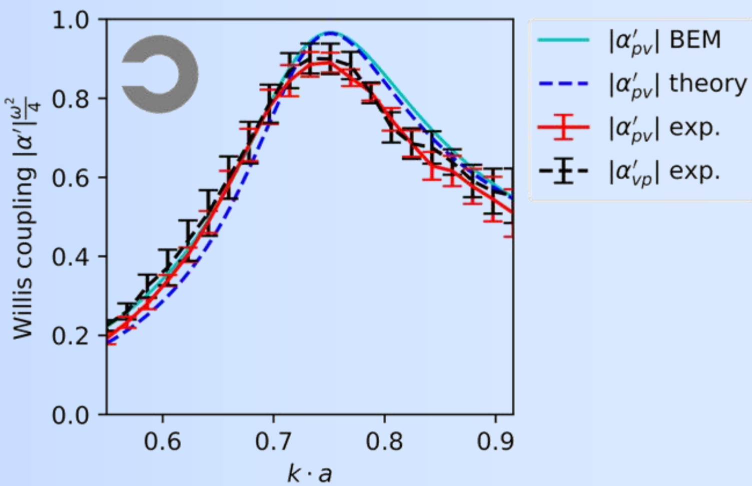


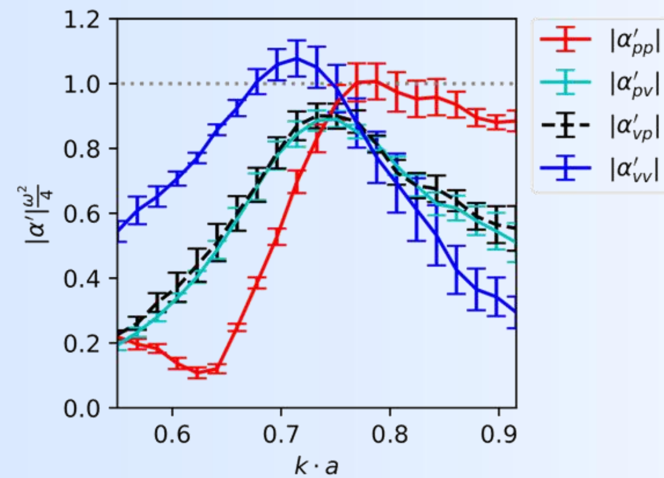
L. Quan, S. Yves, Y. Peng, H. Esfahlani, A. Alù, *Nature Comm.* **12**, 2615 (2021)

Maximum Willis Coupling









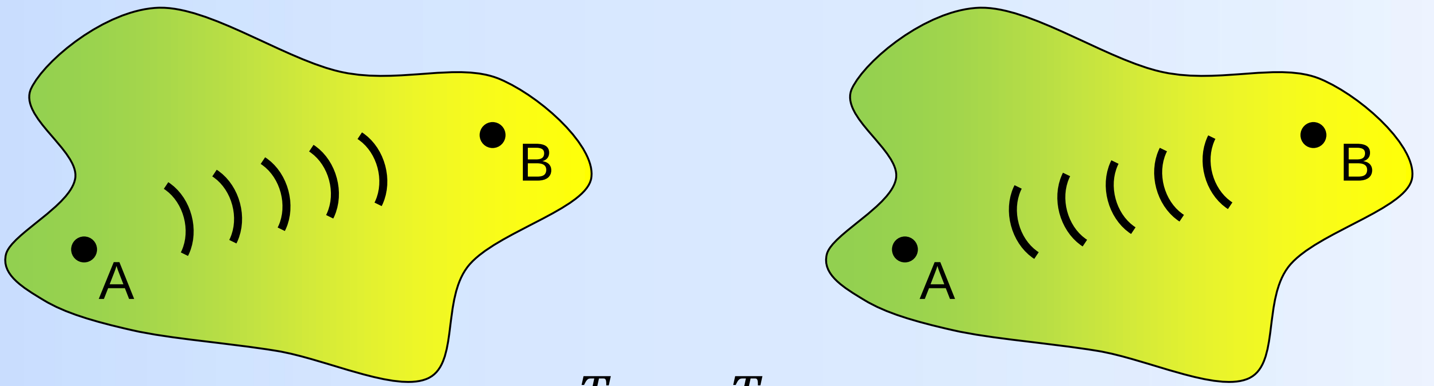
L. Quan, D. Sounas, A. Alù, *Phys. Rev. Lett.* **120**, 254301 (2018)

A. Melnikov, Y. K. Chang, L. Quan, S. Oberst, A. Alù, S. Marburg, D. Powell, *Nature Comm.* **10**, 3148 (2019)

Y. Liu, Z. Liang, J. Zhu, L. Xia, O. Mondain-Monval, T. Brunet, A. Alù, J. Li, *Phys. Rev. X* **9**, 011040 (2019)

Reciprocity in Metamaterials

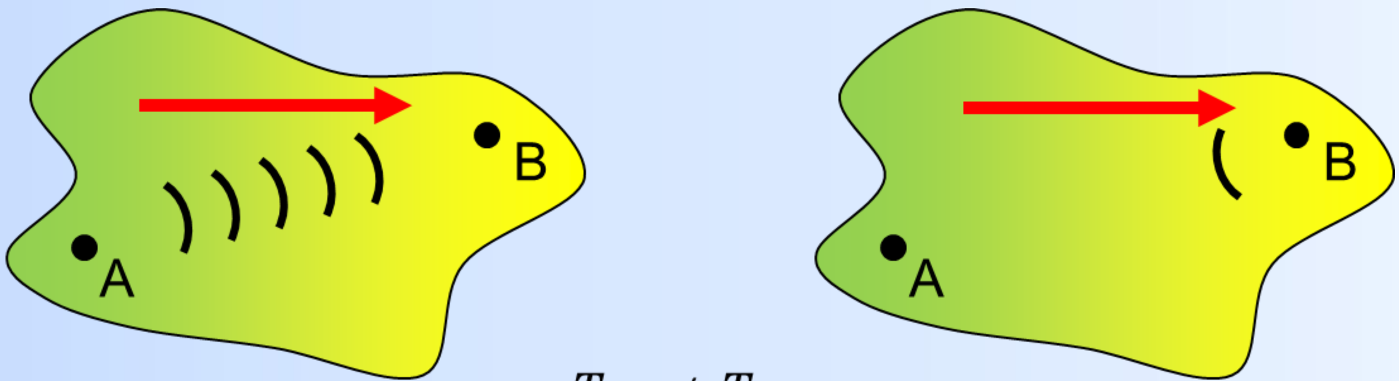
Reciprocity: *symmetry in transmission for opposite propagation directions*



$$T_{\mathrm{BA}} = T_{\mathrm{AB}}$$

Magnet-Free Nonreciprocity

Reciprocity: *symmetry in transmission for opposite propagation directions*



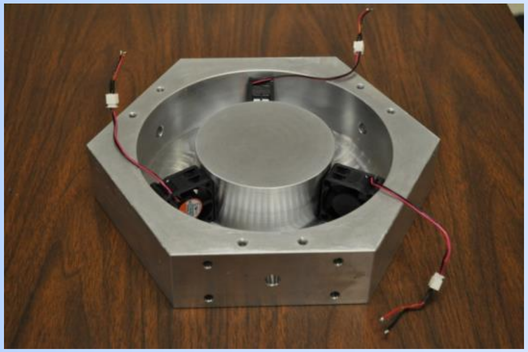
$$T_{BA} \neq T_{AB}$$

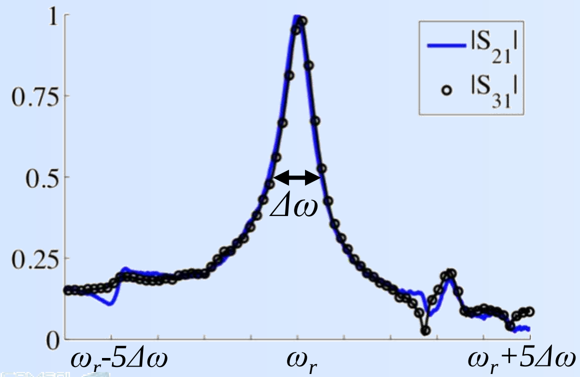
Moving media

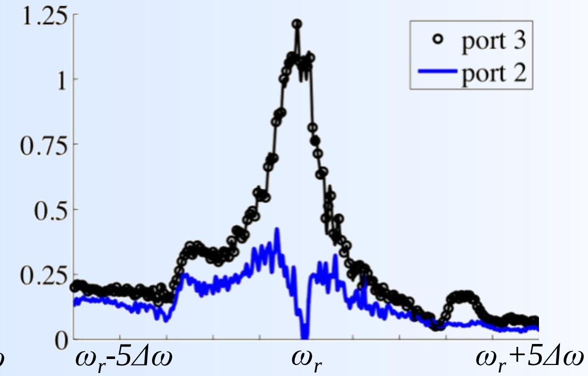
Time-varying materials

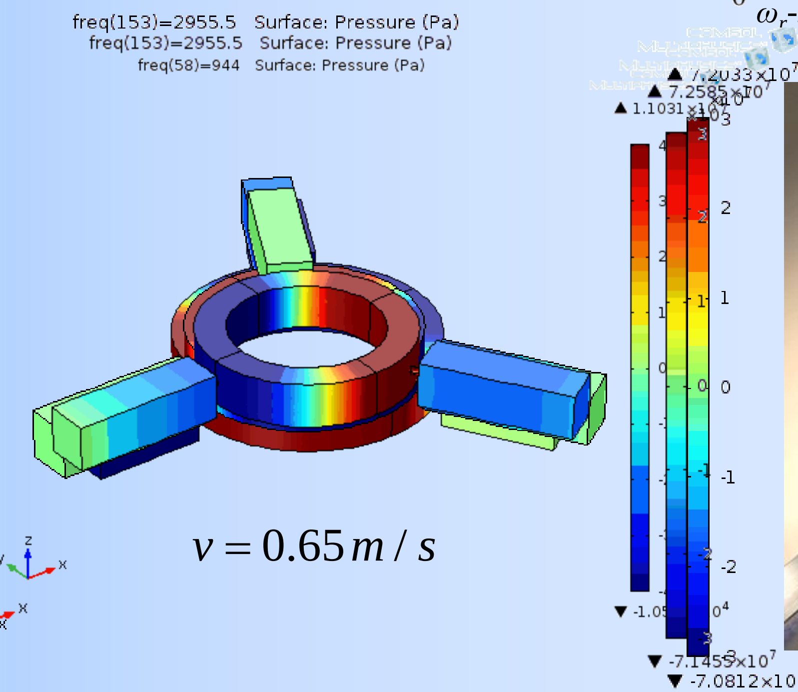
Nonlinearities

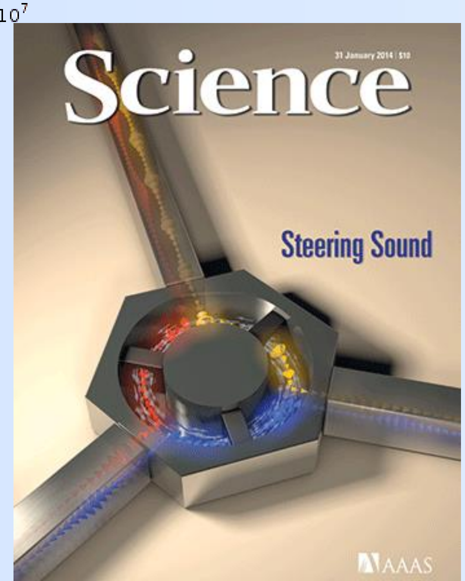
Broken T-Symmetry: Angular-Momentum Bias

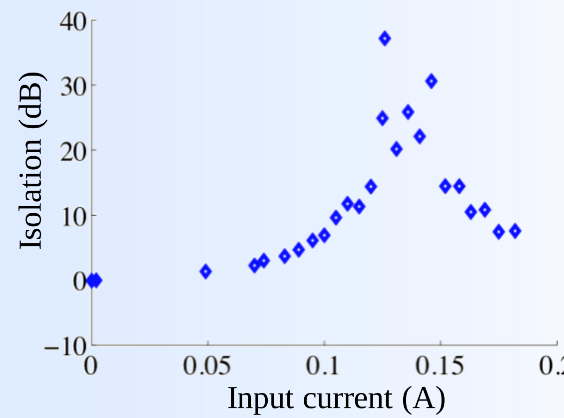






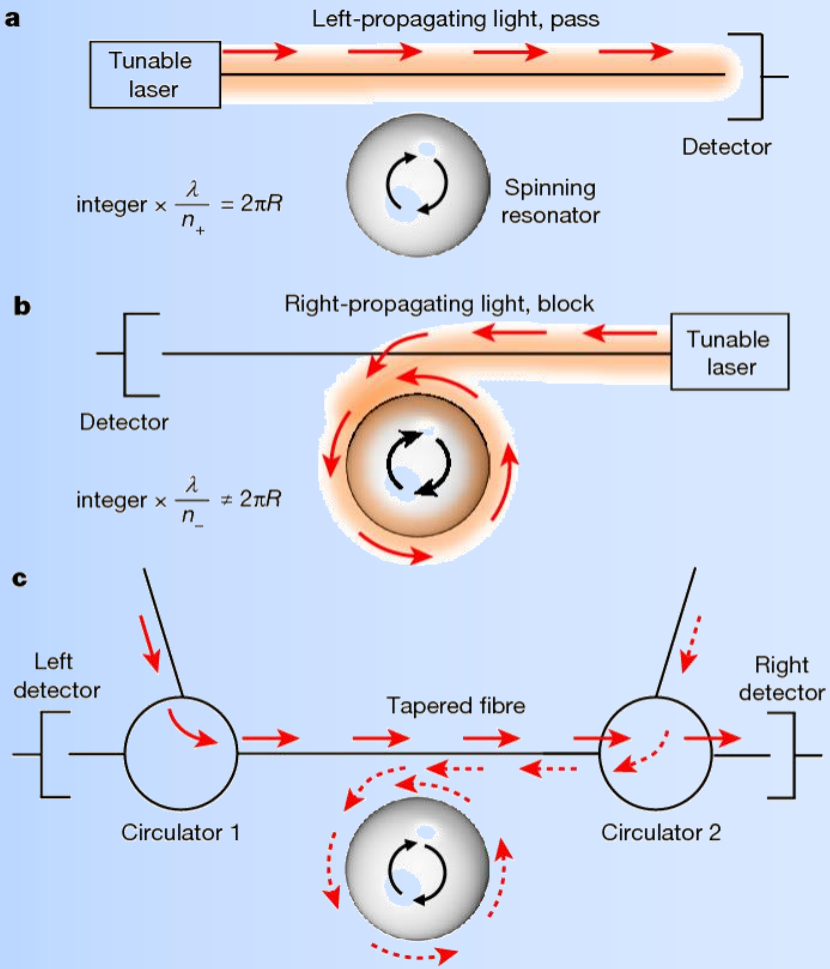


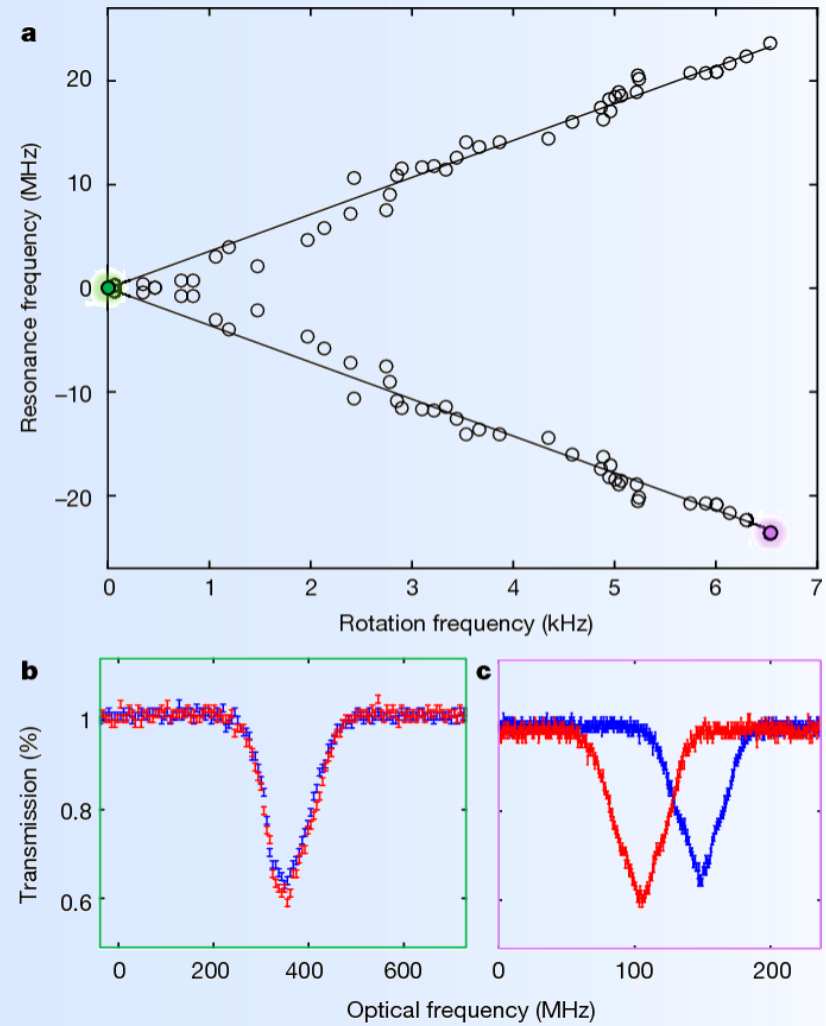




R. Fleury, D. L. Sounas, C. Sieck, M. Haberman, A. Alù, *Science* 343, 516 (2014)

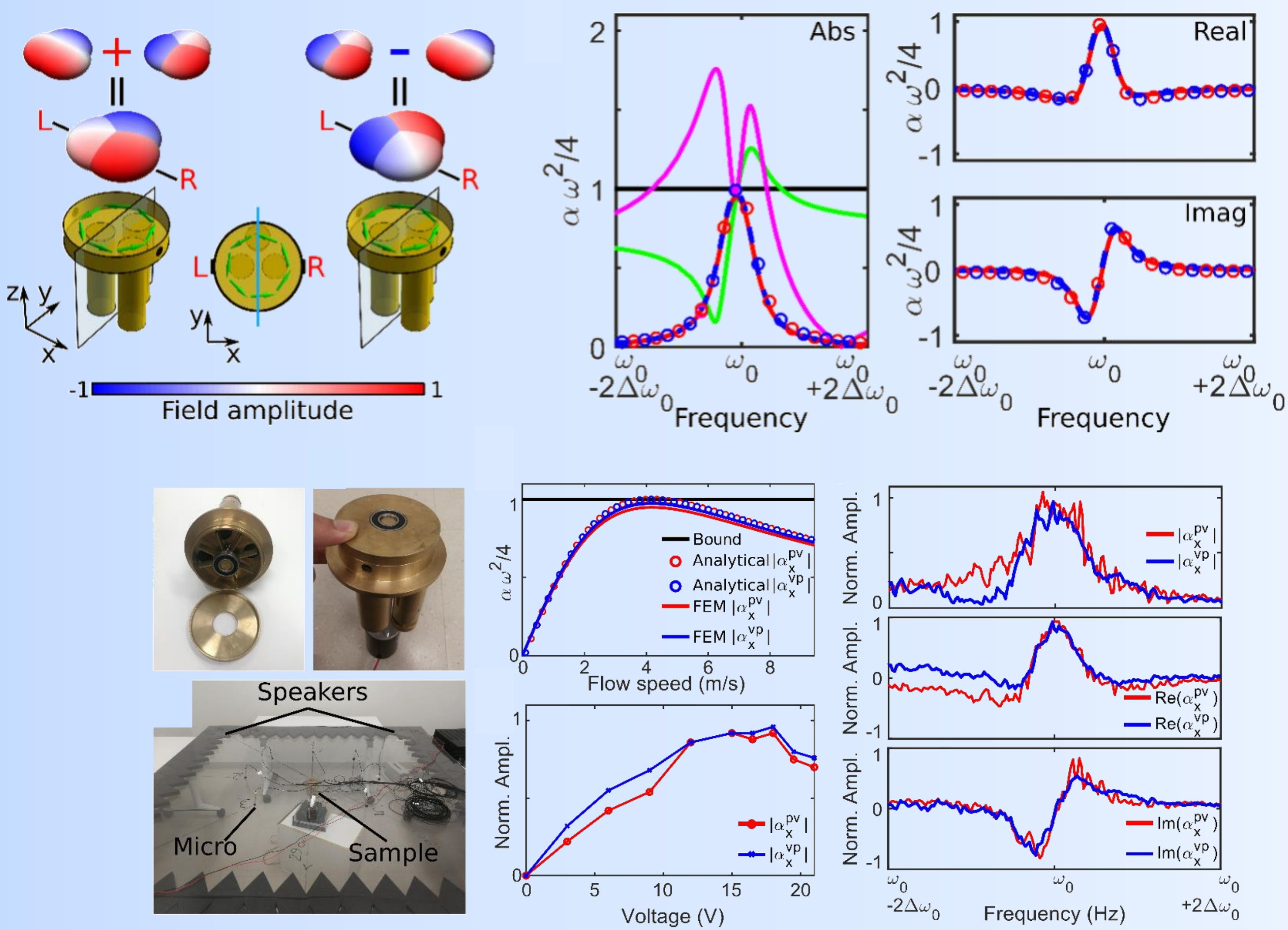
Angular-Momentum Bias in Optics





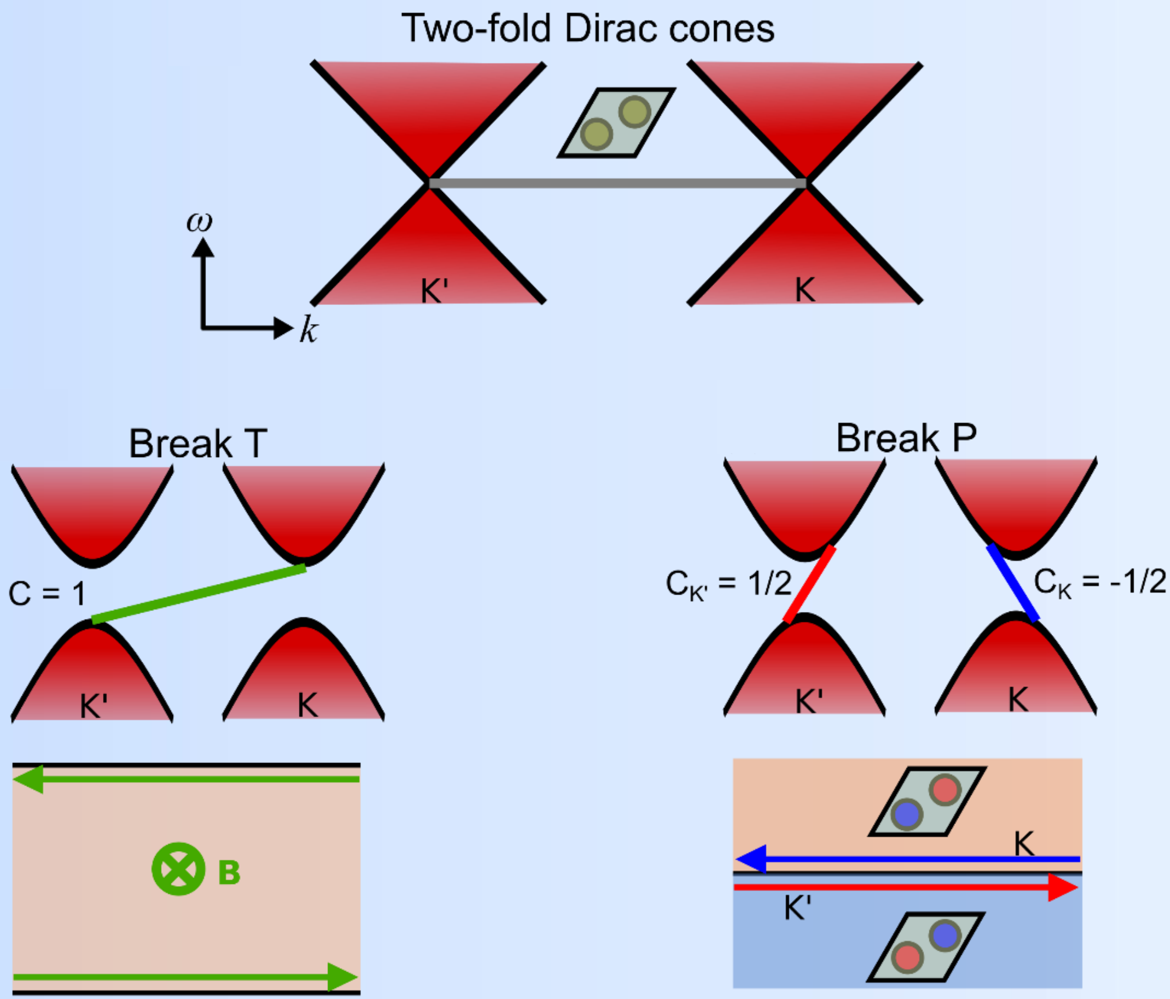
S. Maayani, et al., *Nature* **558**, 569 (2018)

Odd-Symmetric Willis Coupling



L. Quan, S. Yves, Y. Peng, H. Esfahlani, A. Alù, *Nature Comm.* **12**, 2615 (2021)

Topological Phases of Matter



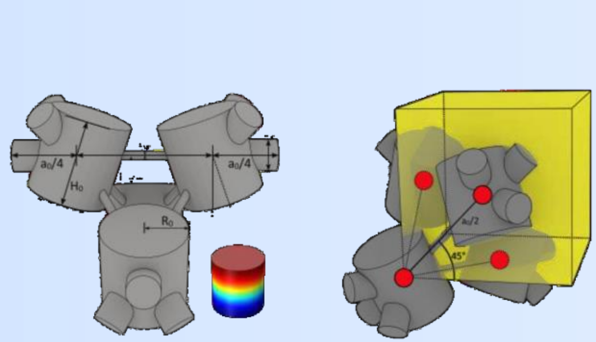
S. Yves, X. Ni, A. Alù, *Annals NY Academ. Sci.* **1517**, 63 (2022)

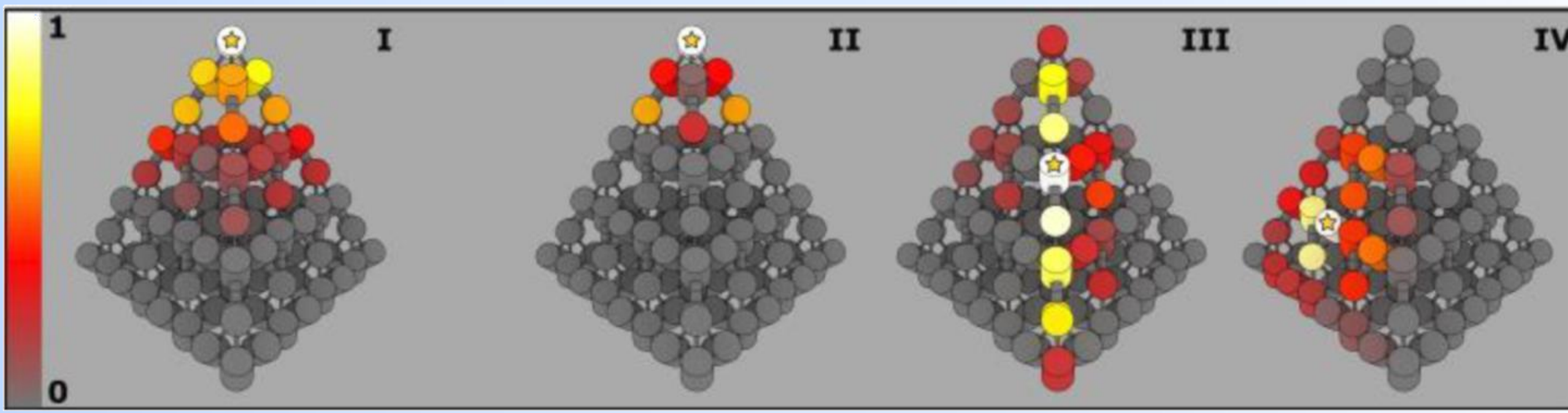
Topological Sound Based on Generalized Chirality

$$\hat{\Gamma}_3 \hat{H}_0 \hat{\Gamma}_3^{-1} = \hat{H}_1$$

$$\hat{\Gamma}_3 \hat{H}_1 \hat{\Gamma}_3^{-1} = \hat{H}_2$$

$$\hat{H}_0 + \hat{H}_1 + \hat{H}_2 = 0$$

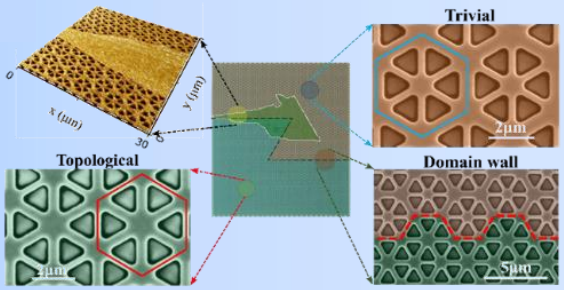




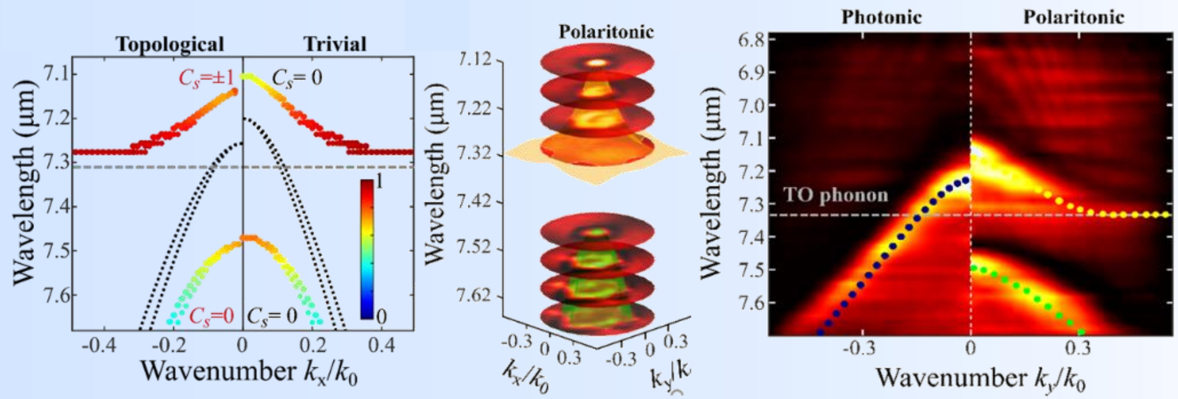
X. Ni, M. Li, M. Weiner, A. Alù, A. B. Khanikaev, *Nature Comm.* **11**, 2108 (2020)

Topological Phonon Polaritons

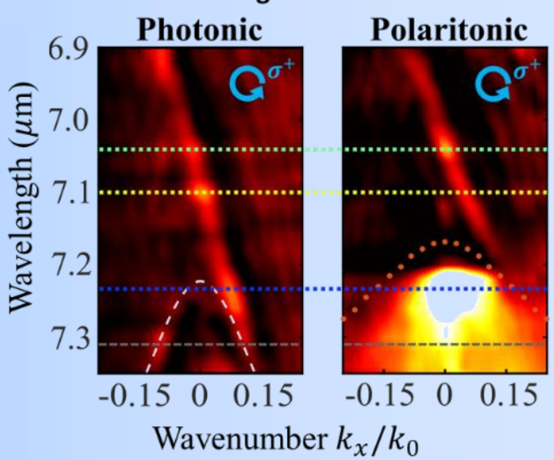
Topological domain wall with 15nm hBN



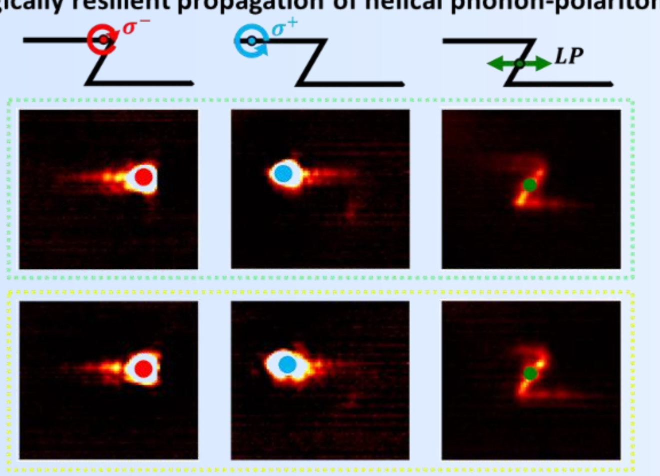
Experiment + Theory – IR topological bulk phonons polaritons



Helical edge states in mid-IR

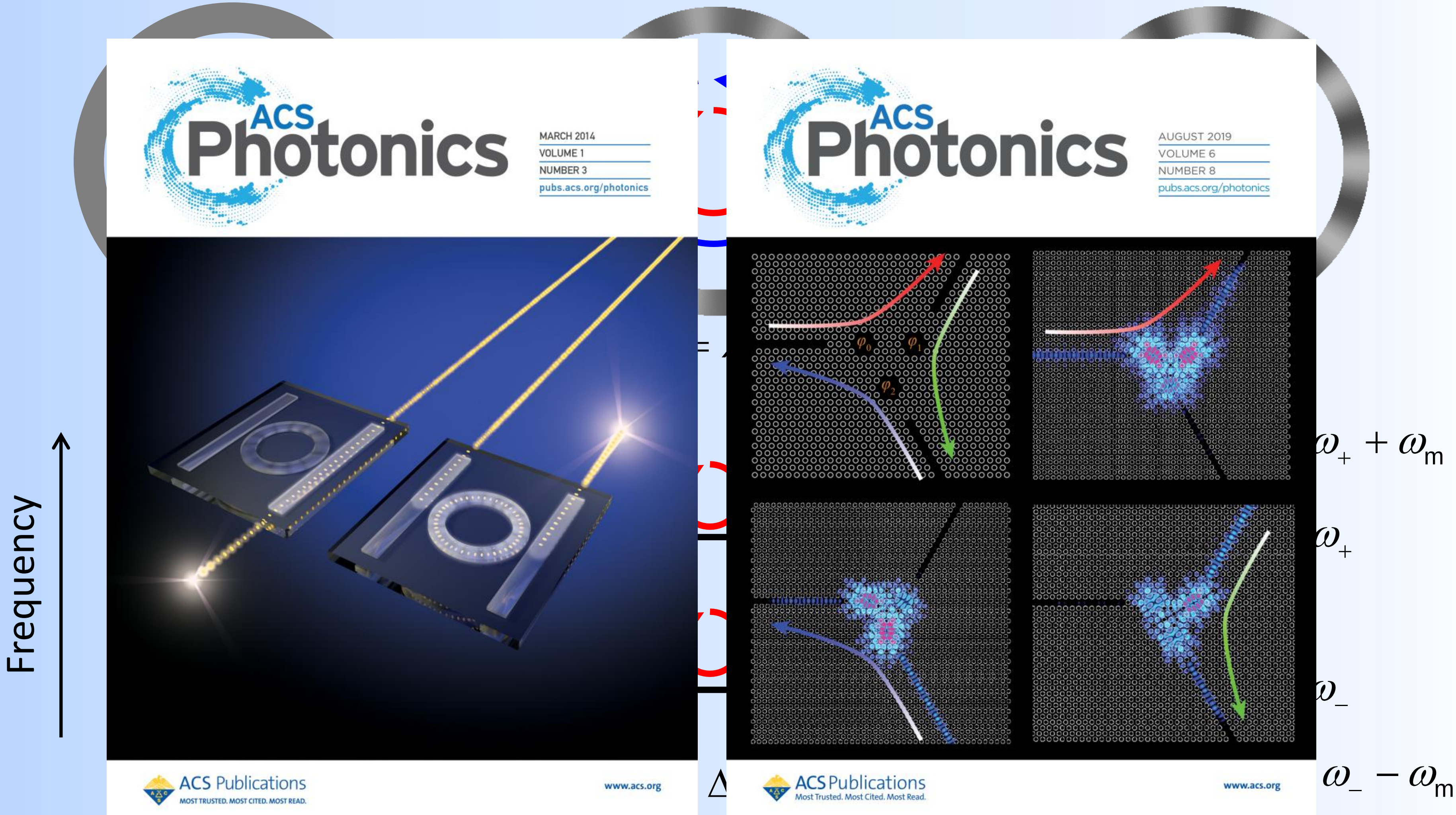


Topologically resilient propagation of helical phonon-polaritons



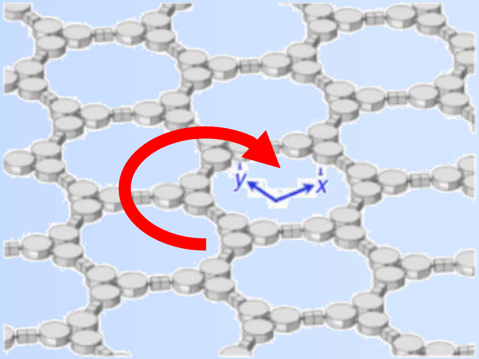
S. Guddala, et al., *Science* **374**, 225 (2021)

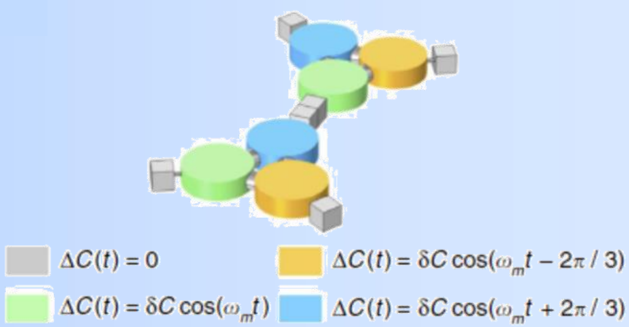
Synthetic Angular Momentum with Time Modulation

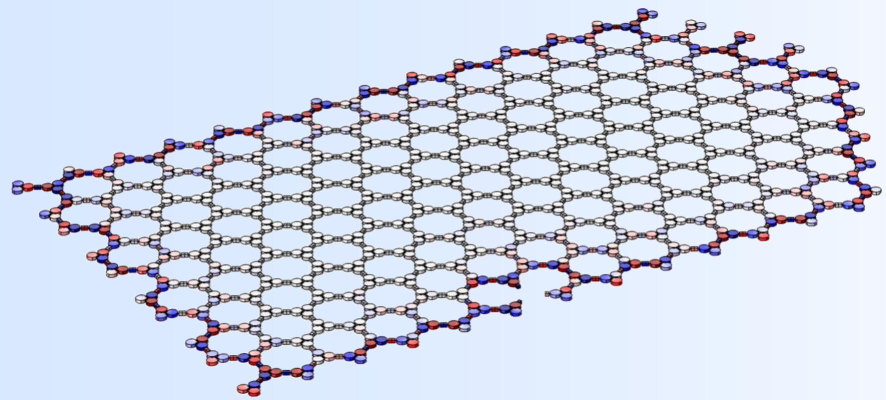


D. Sounas, A. Alù, *ACS Photonics* **1**, 198 (2014)
A. Mock, D. Sounas, A. Alù, *ACS Photonics* **6**, 2056 (2019)

FLOQUET TOPOLOGICAL INSULATORS

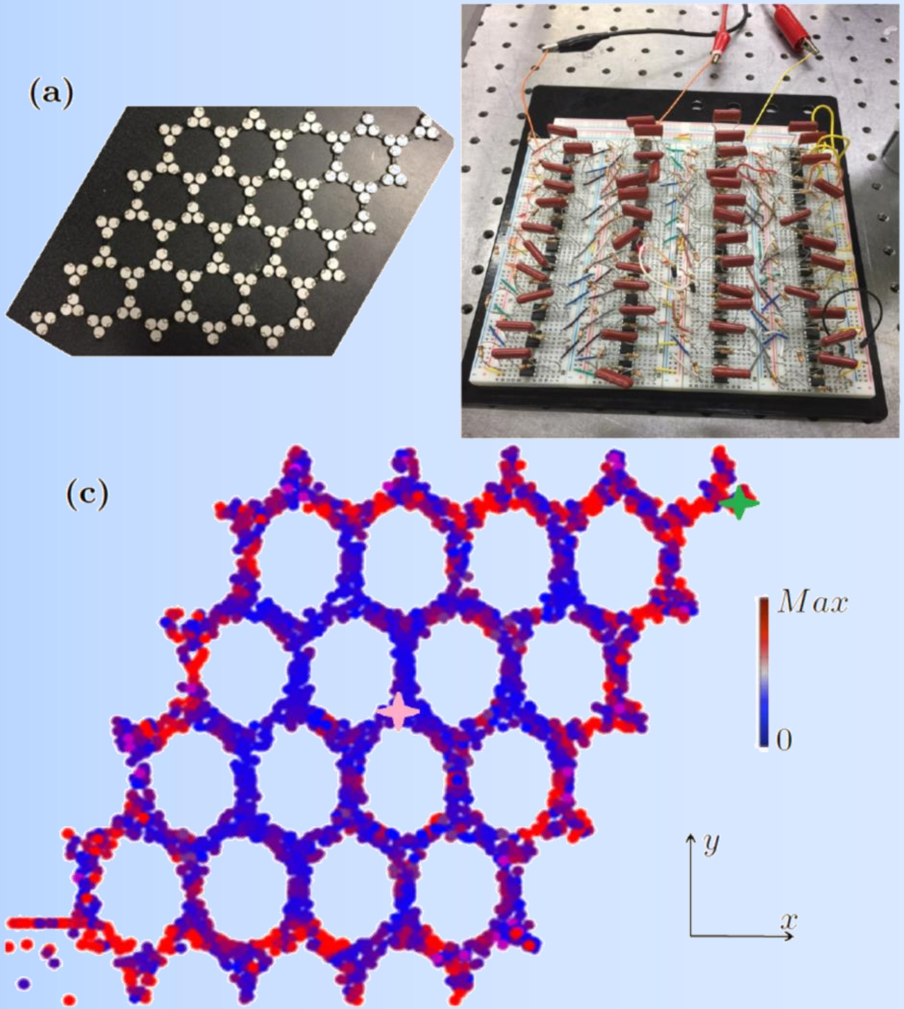


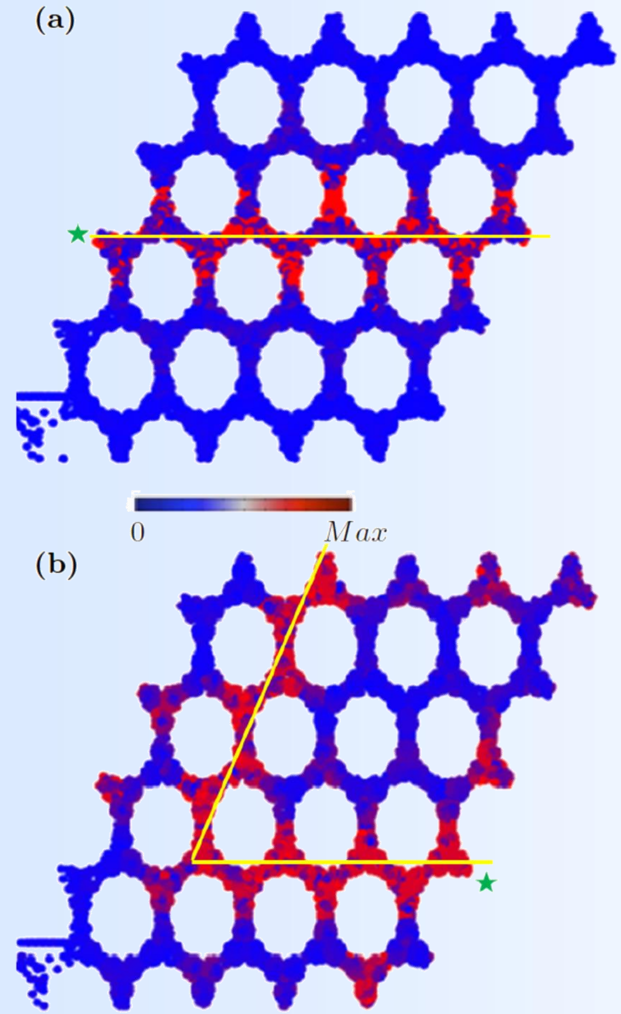




A. B. Khanikaev, R. Fleury, H. Mousavi, A. Alù, *Nature Communications* **6**, 8260 (2015)
R. Fleury, A. B. Khanikaev, A. Alù, *Nature Communications*, **7**, 11744 (2016)

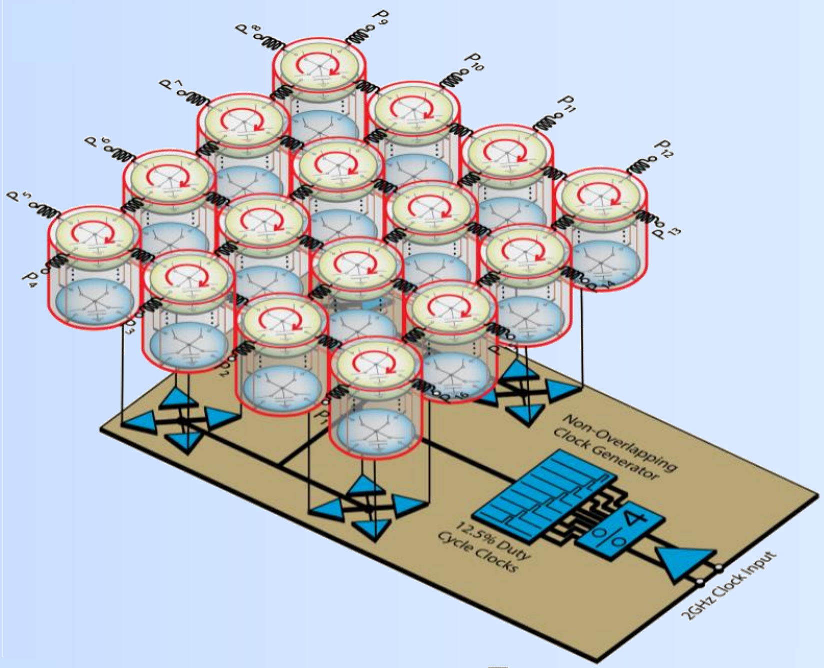
Floquet Topological Insulators for Elastic Waves

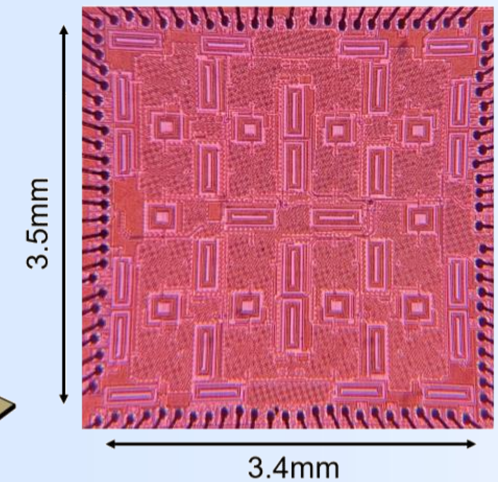


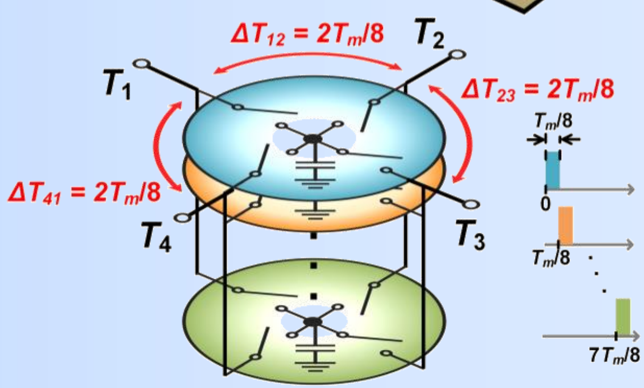


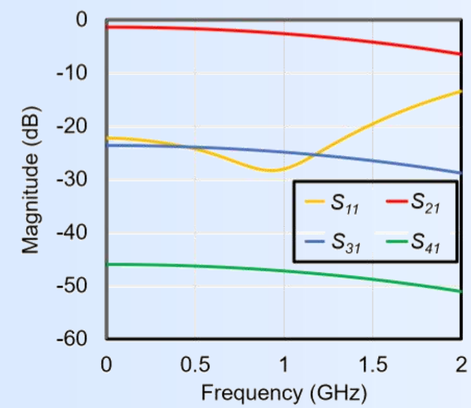
A. Ardabi, M. Leamy, A. Alù, *Science Advances* **6**, eaba8656 (2020)

CMOS Ultra-Wideband Topological Insulators



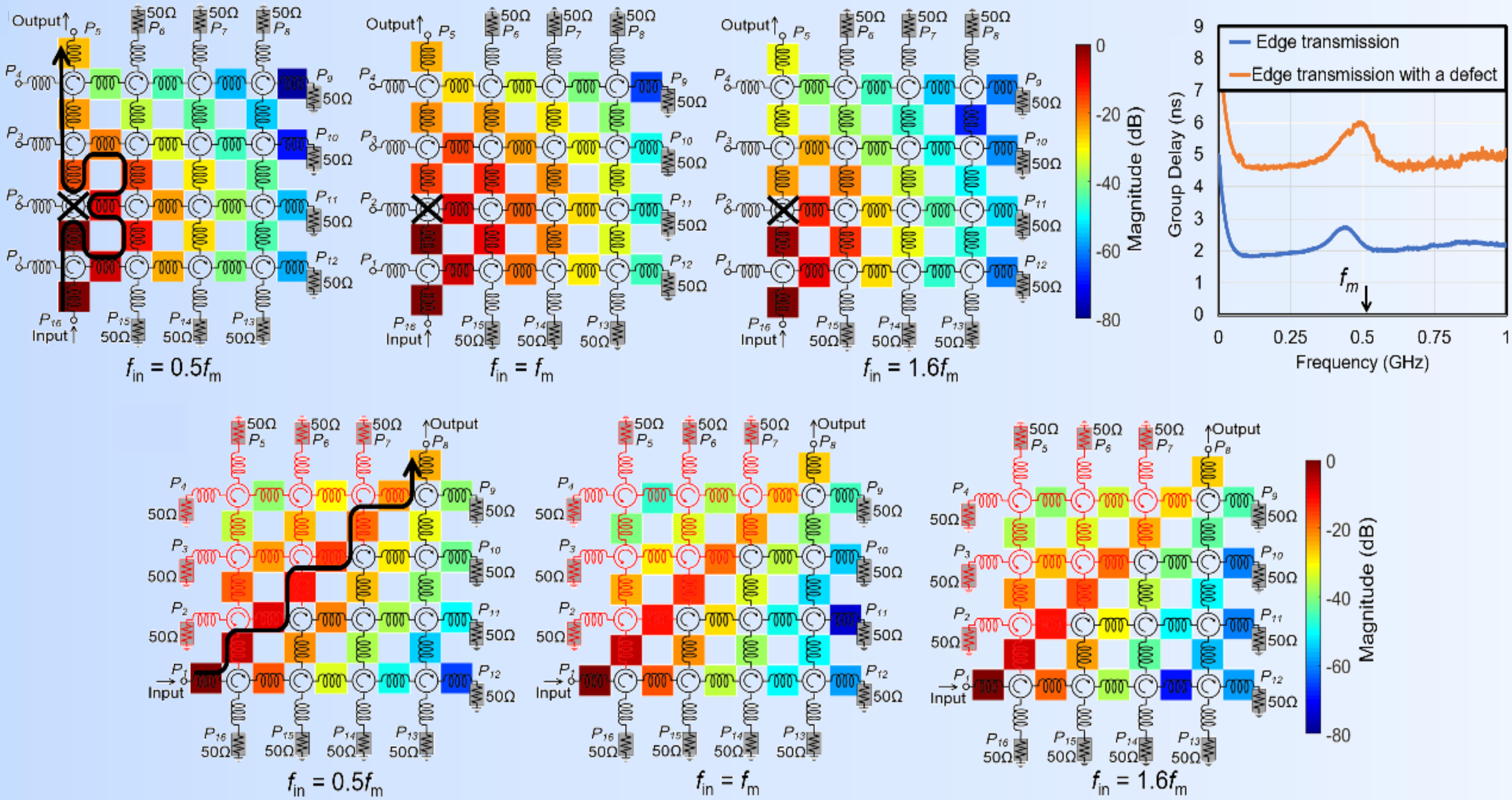






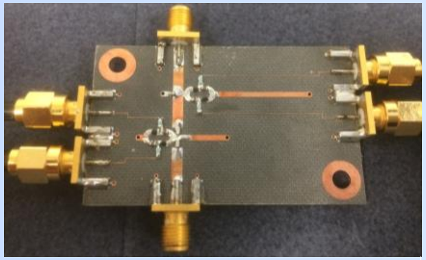
A. Nagulu, et al., *Nature Electronics* **5**, 300 (2022)

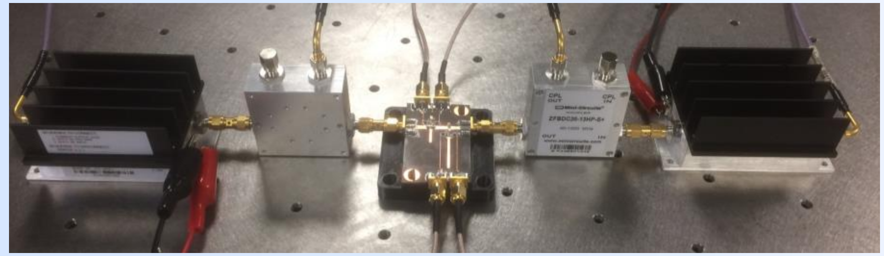
CMOS Ultra-Wideband Topological Insulators

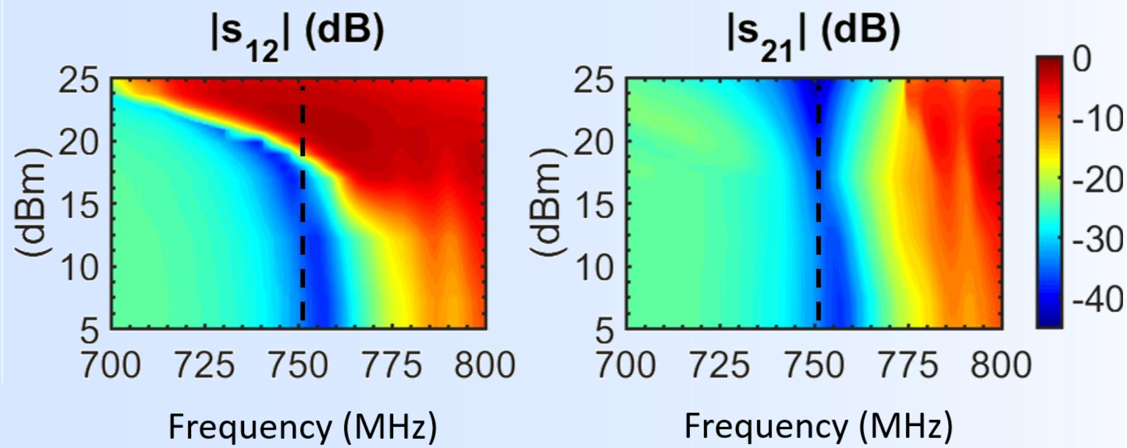


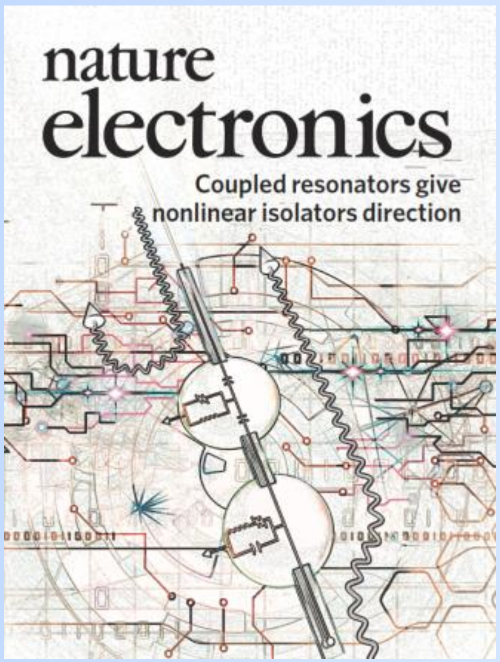
A. Nagulu, et al., *Nature Electronics* **5**, 300 (2022)

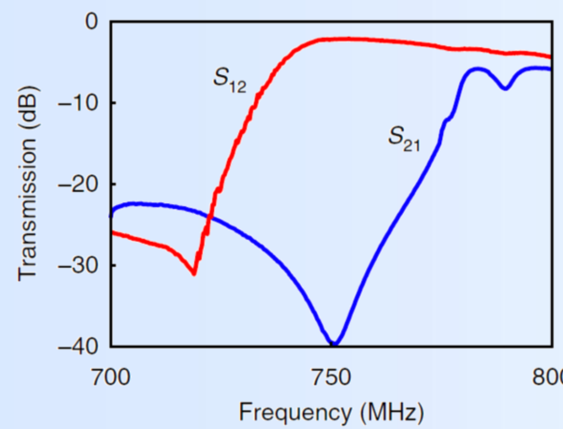
Nonreciprocity Based on Nonlinearities and Asymmetry





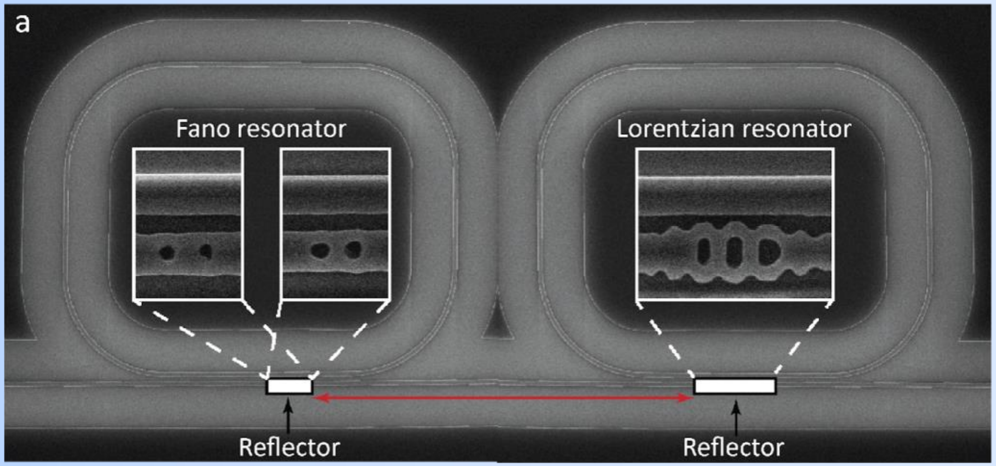


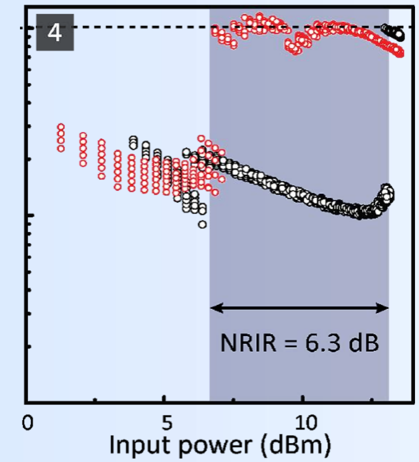


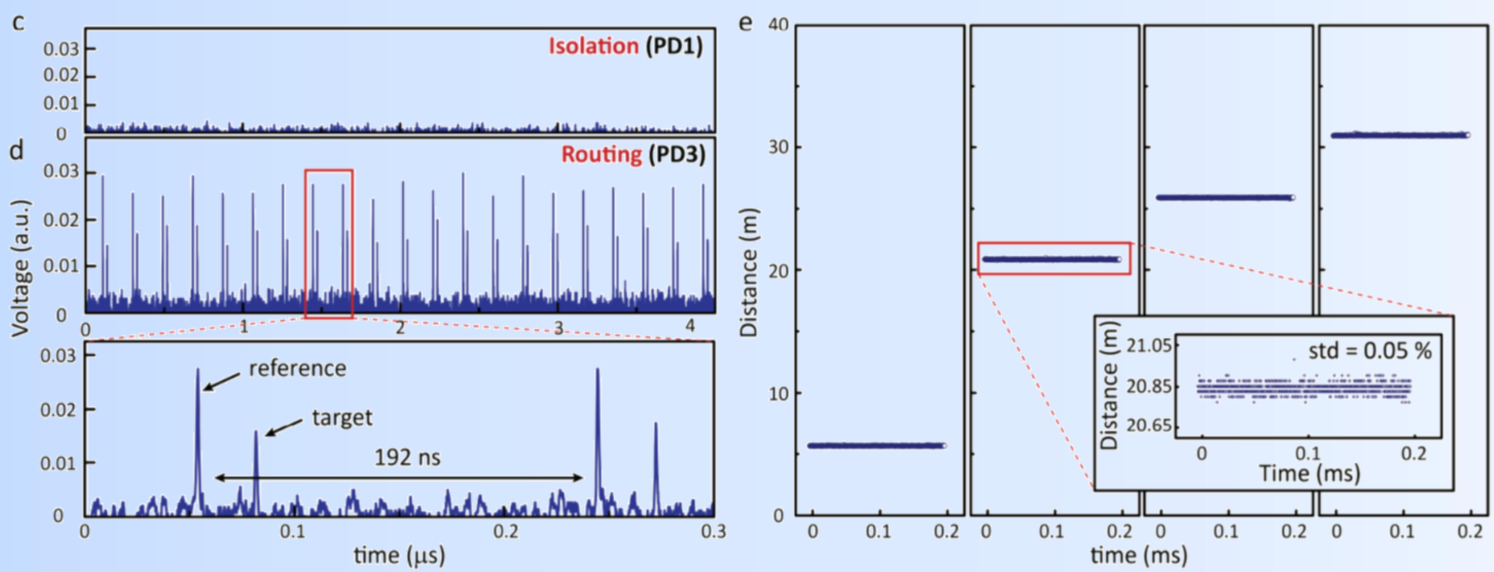


D. L. Sounas, J. Soric, and A. Alù
Nature Electron. **1**, 113 (2018)

Non-Reciprocal Light Propagation with Non-Linearities

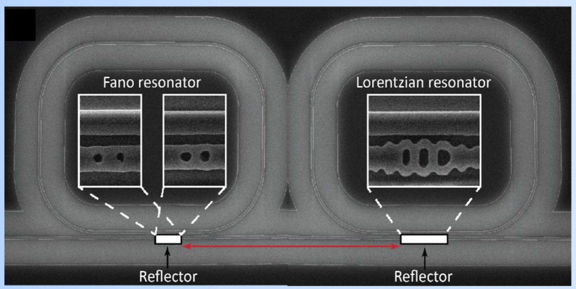


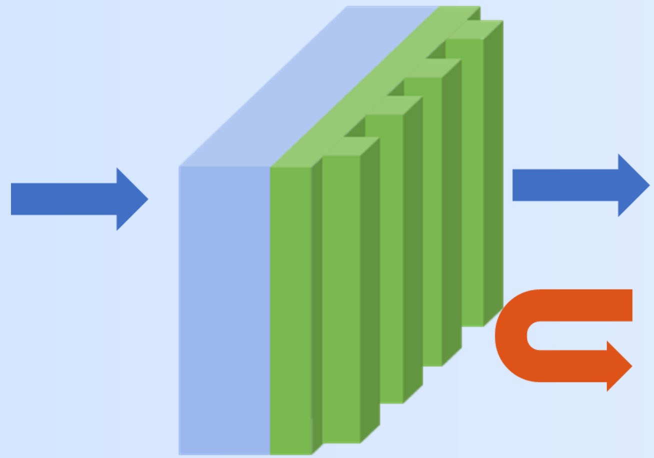


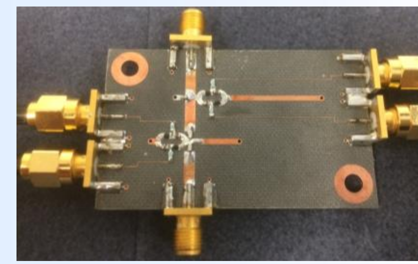


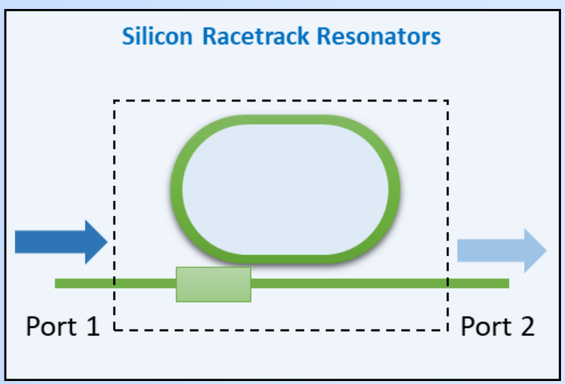
K. Y. Yang, J. Skarda, M. Cotrufo, et al. *Nature Photonics* **14**, 369 (2020)

Bias-Free Nonreciprocal Response with Nonlinearity

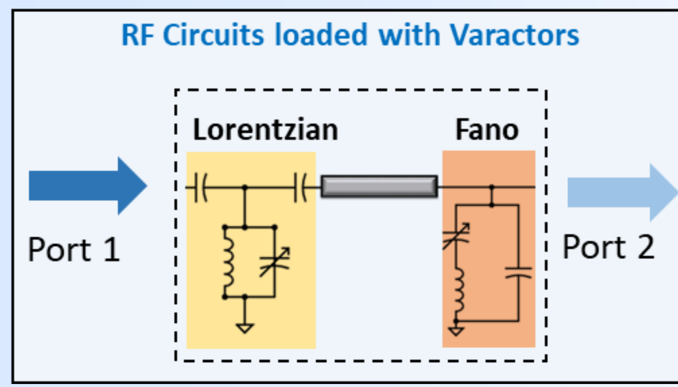




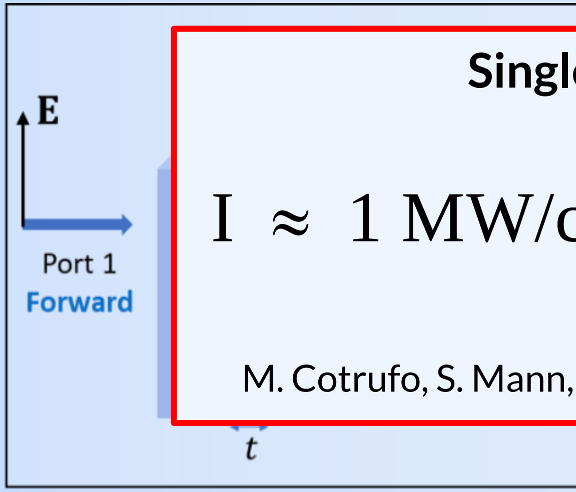




Nature Photonics (2020)



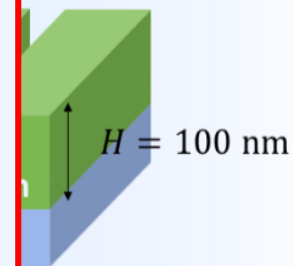
Nature Electronics (2018)



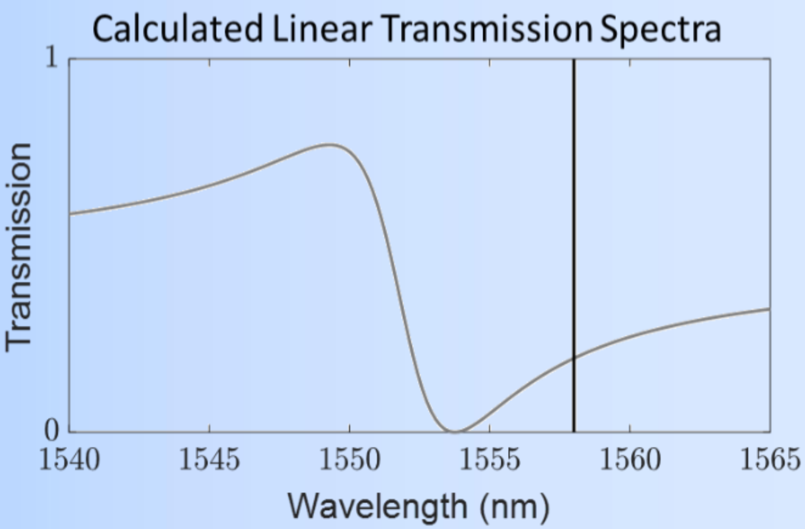
Single-mode Si metasurfaces

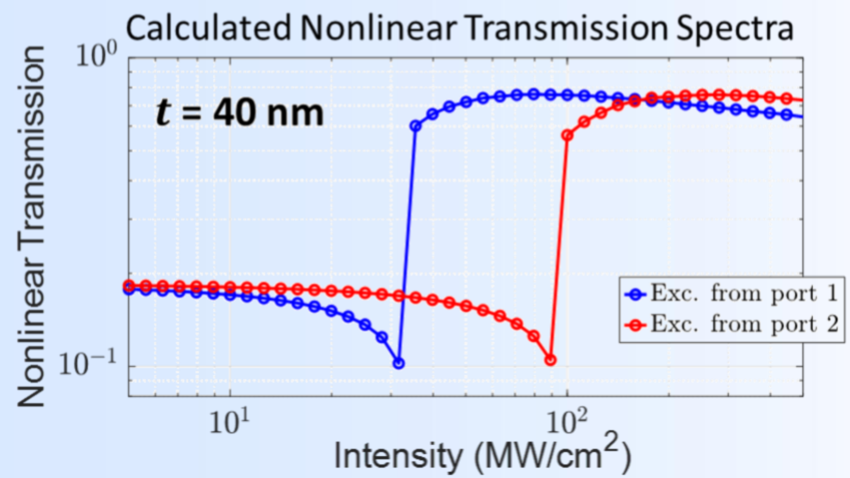
$$I \approx 1\ \mathrm{MW/cm^2} \times \frac{\gamma_{tot}[\mathrm{GHz}] \cdot H_{eff}[\mu\mathrm{m}]}{Q}$$

M. Cotrufo, S. Mann, H. Moussa, A. Alù, *IEEE T-MTT* **69**, 3569 (2021)

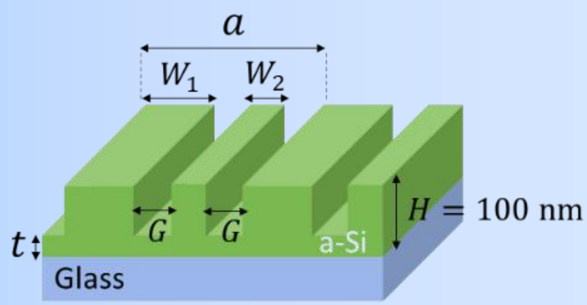


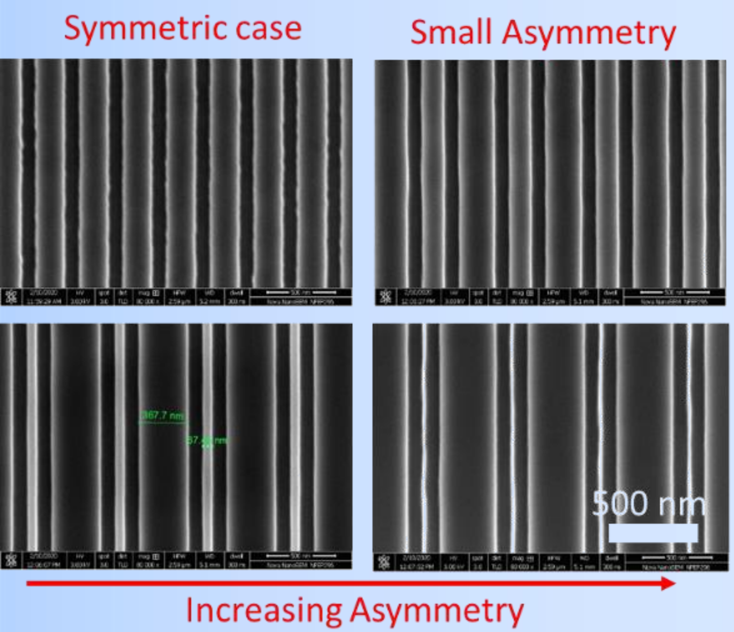
Assuming
- instantaneous Kerr nonlinearity, $\chi^{(3)} = 2.8 \cdot 10^{-18} m^2 / V^2$
- CW excitation

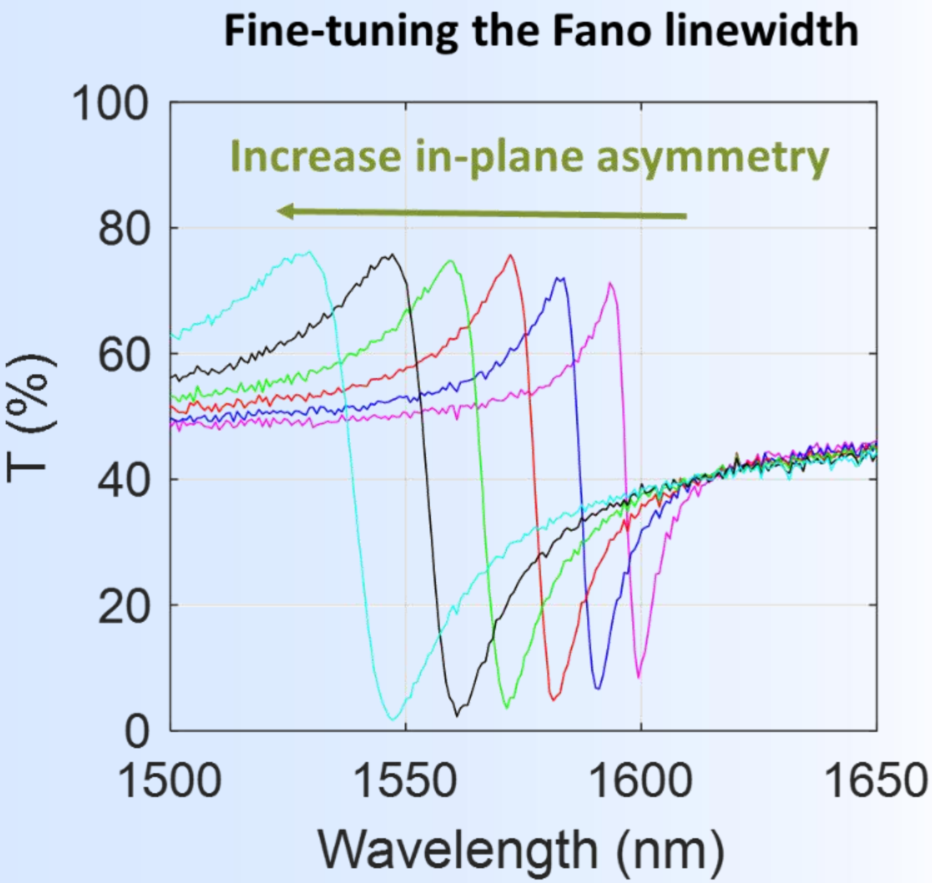




Bias-Free Nonreciprocal Response with Nonlinearity

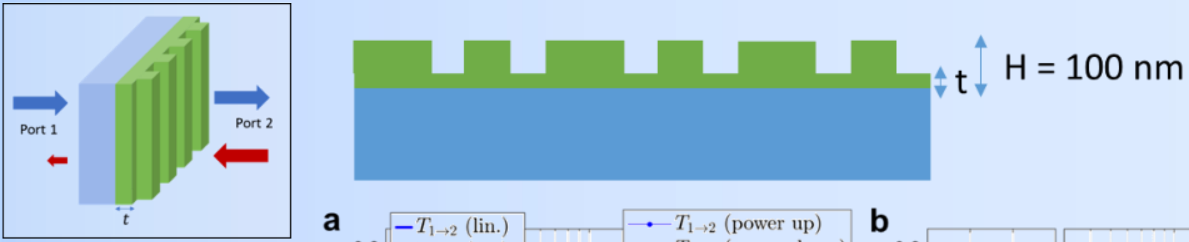




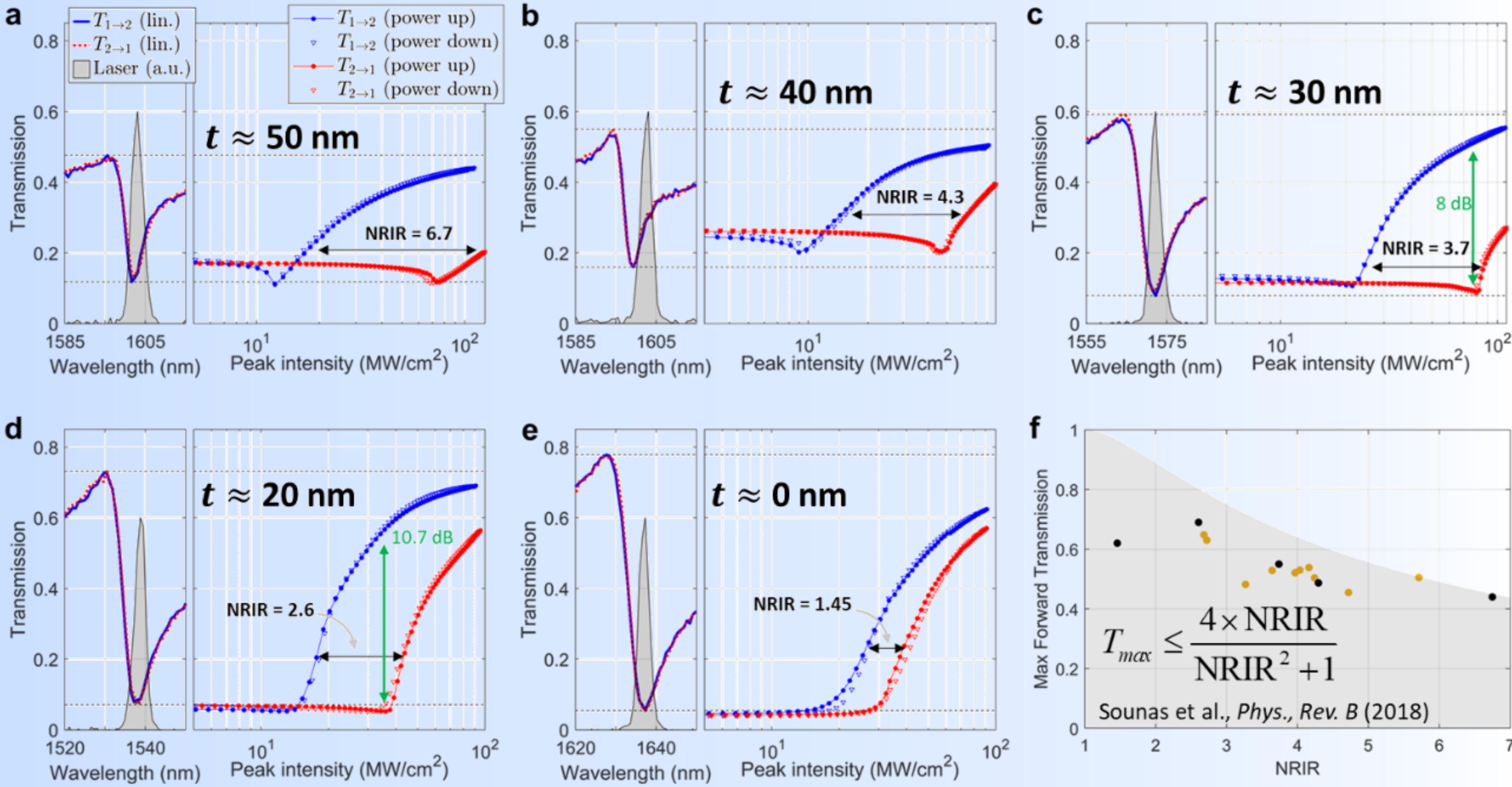


M. Cotrufo, et al., *Nature Photonics* **18**, 81 (2024)

Bias-Free Nonreciprocal Q-BIC Metasurfaces

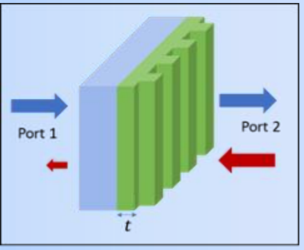


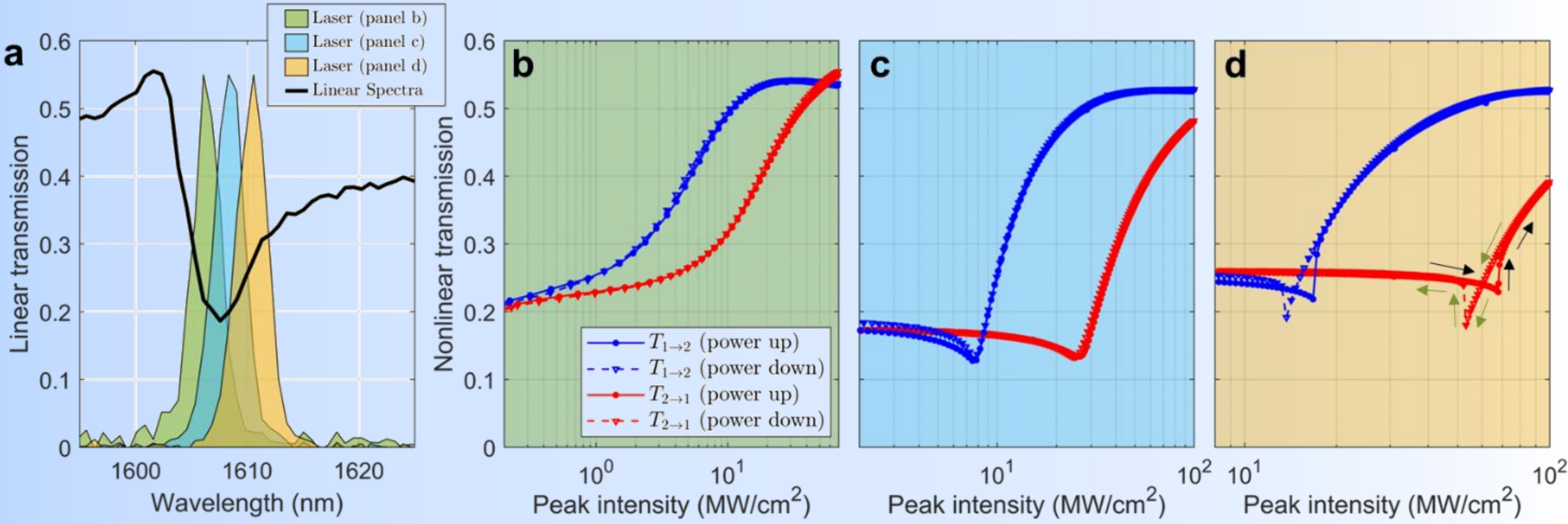
$$\text{NRIR} = \frac{I_2}{I_1}$$



M. Cotrufo, et al., *Nature Photonics* **18**, 81 (2024)

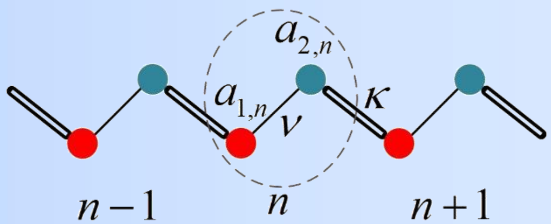
Bias-Free Nonreciprocal Q-BIC Metasurfaces



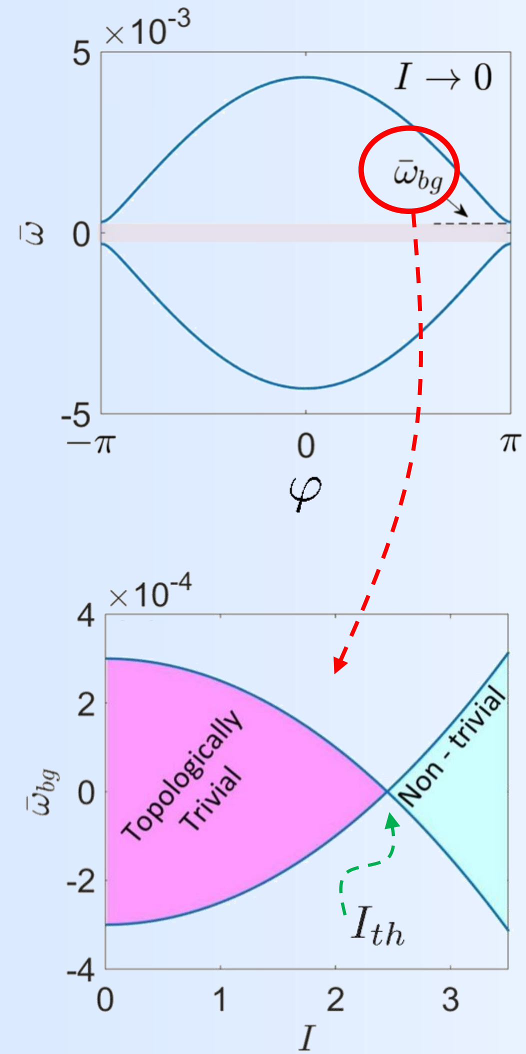


M. Cotrufo, et al., *Nature Photonics* **18**, 81 (2024)

Nonlinearity-Induced Topological Transitions



$$i\frac{d\Psi_n}{dt} = \mathbf{\Omega}\Psi_n + \mathbf{K}_m\Psi_{n-1} + \mathbf{K}_p\Psi_{n+1}$$

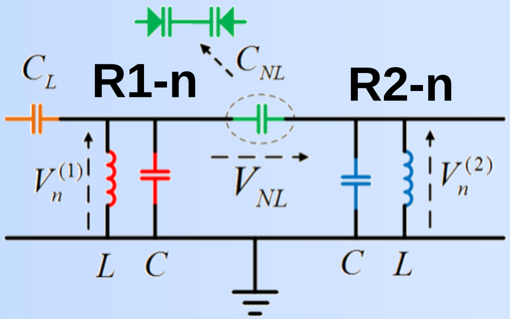


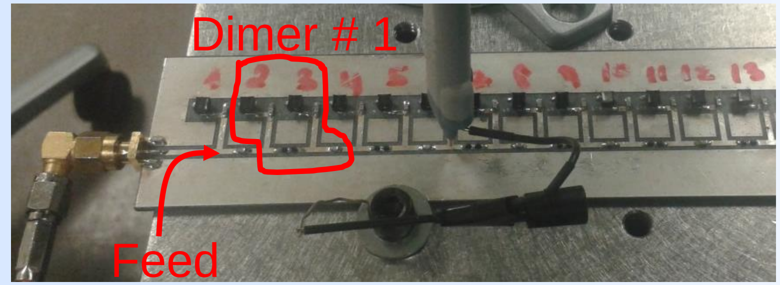
$\nu > \kappa_0$

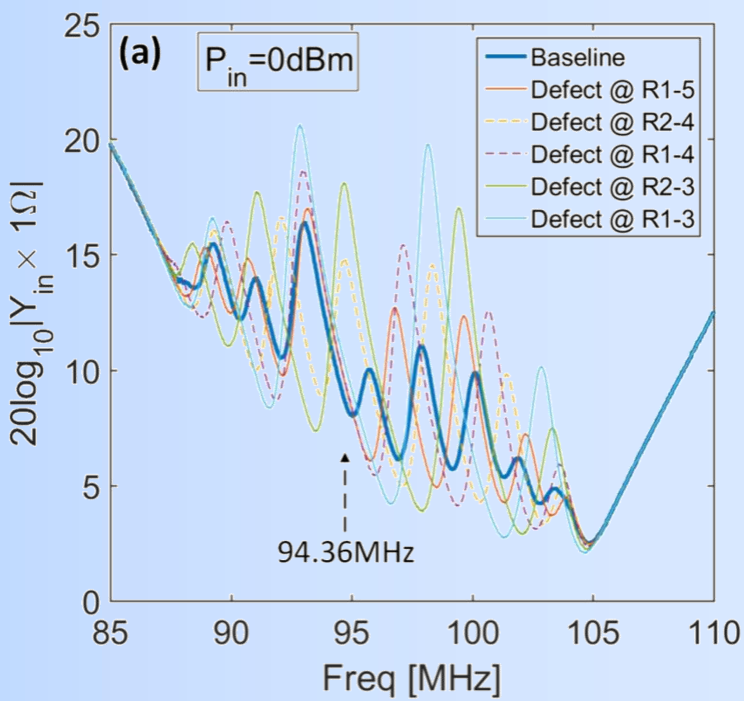
$$\bar{\omega}_{bg} = \pm|\nu - \kappa_0 - \alpha I^2| \Rightarrow I_{th} = \sqrt{\frac{\nu - \kappa_0}{\alpha}}$$

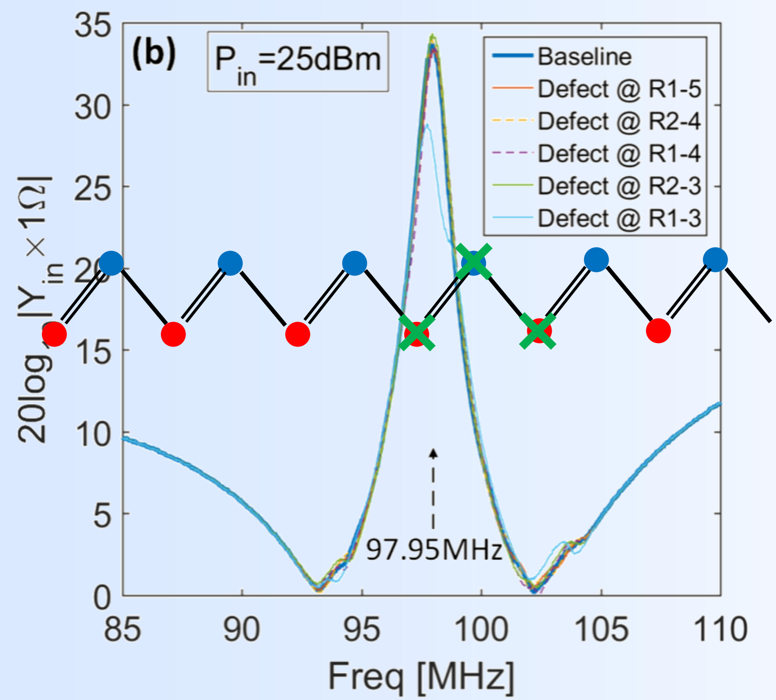
Y. Hadad, A. B. Khanikaev, and A. Alù, *Phys. Rev. B* **93**, 155112 (2016)

NONLINEARITY-INDUCED TOPOLOGICAL ORDER



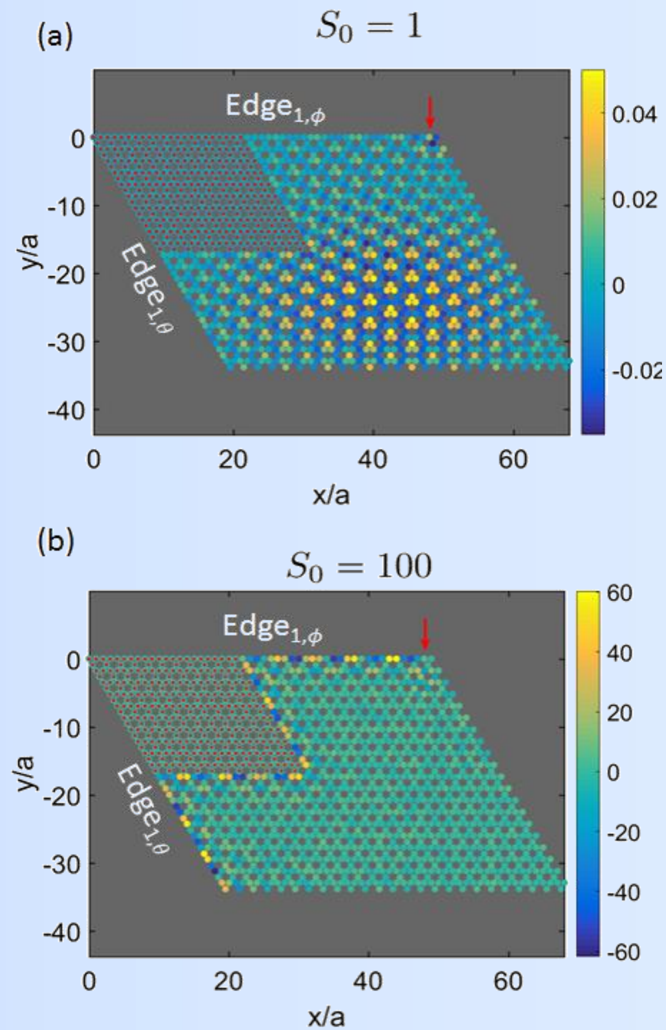






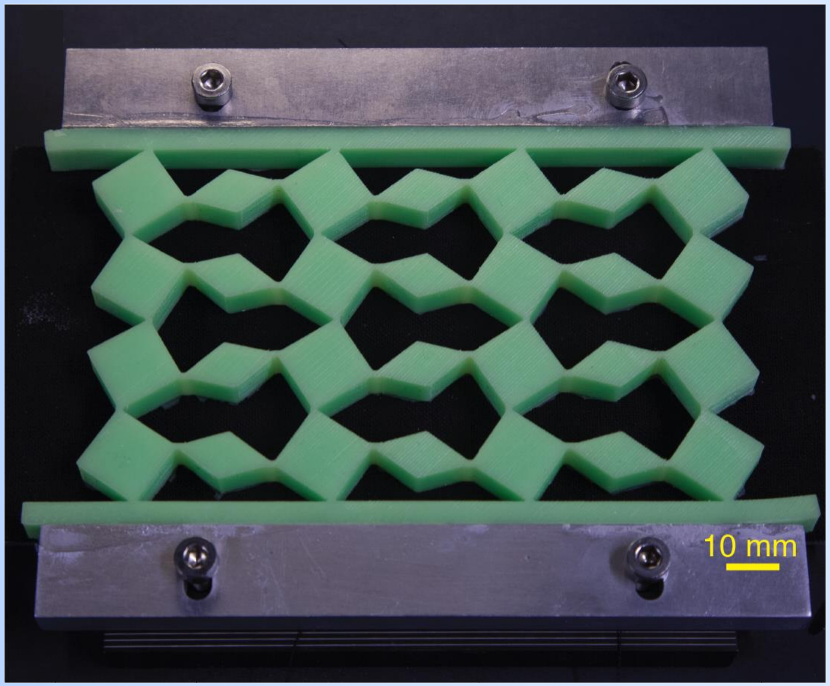
Y. Hadad, A. B. Khanikaev, and A. Alù, *Nature Electronics* **1**, 178 (2018)

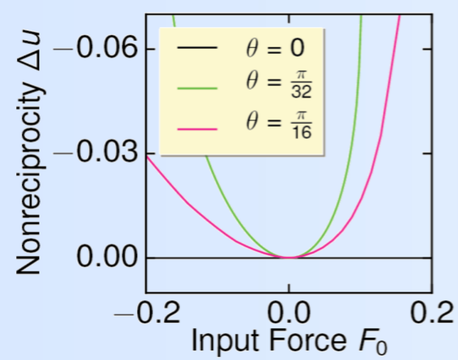
Topological Transition Triggered by Signal Intensity

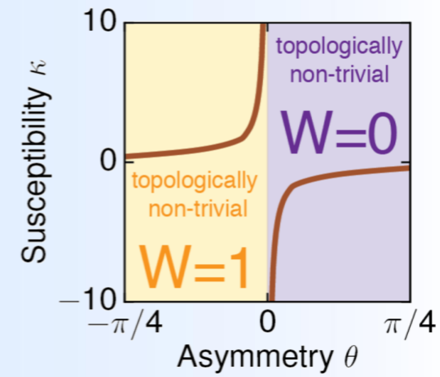


G. D'Aguanno, Y. Hadad, D. A. Smirnova, X, Ni, A. Khanikaev, A. Alù, *Phys. Rev. B* **100**, 214310 (2019)

Topological Mechanics







C. Coulais, D. Sounas, A. Alù, *Nature* **542**, 461 (2017)

PT-Symmetry & Non-Hermitian Hamiltonians

Observables in quantum mechanics are represented by Hermitian operators known to exhibit real eigenvalues.

Should a Hamiltonian be Hermitian in order to have real eigenvalues?

VOLUME 80, NUMBER 24 PHYSICAL REVIEW LETTERS 15 JUNE 1998

Real Spectra in Non-Hermitian Hamiltonians Having $\mathcal{PT}$ Symmetry

Carl M. Bender[1] and Stefan Boettcher[2,3]
[1] *Department of Physics, Washington University, St. Louis, Missouri 63130*
[2] *Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*
[3] *CTSPS, Clark Atlanta University, Atlanta, Georgia 30314*
(Received 1 December 1997; revised manuscript received 9 April 1998)

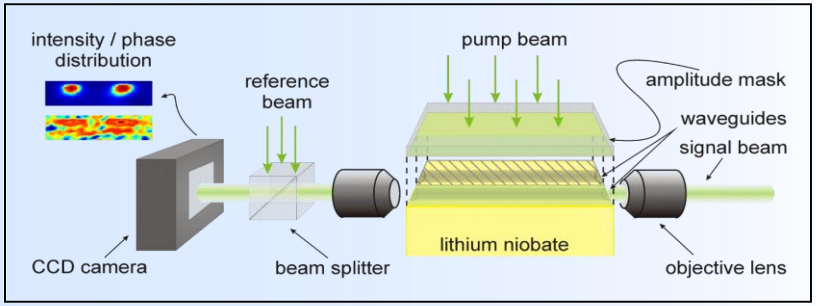
The condition of self-adjointness ensures that the eigenvalues of a Hamiltonian are real and bounded below. Replacing this condition by the weaker condition of $\mathcal{PT}$ symmetry, one obtains new infinite classes of complex Hamiltonians whose spectra are also real and positive. These $\mathcal{PT}$ symmetric theories may be viewed as analytic continuations of conventional theories from real to complex phase space. This paper describes the unusual classical and quantum properties of these theories. [S0031-9007(98)06371-6]

Parity-time (PT) symmetric Hamiltonians share common eigenfunctions with the PT operator. As a result they can exhibit entirely real spectra!

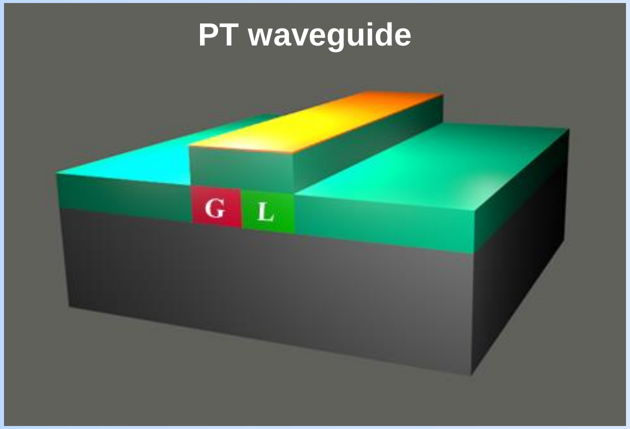
Parity-Time Symmetry

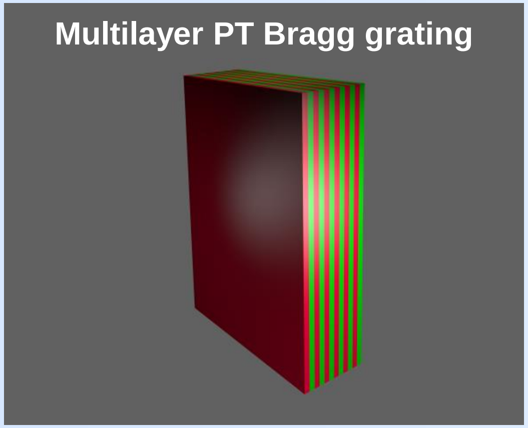
$$n(\mathbf{r}) = n^*(-\mathbf{r})$$

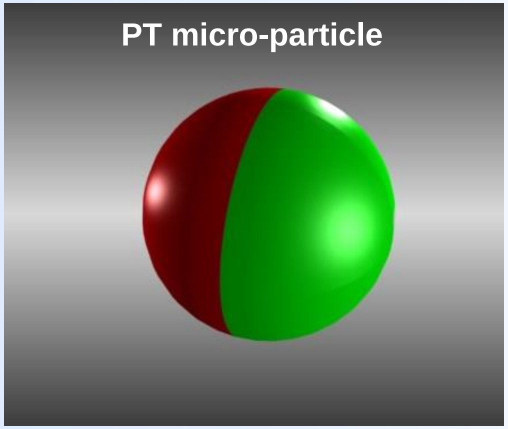
$$n_R(-\mathbf{r}) = +n_R(\mathbf{r})$$
$$n_I(-\mathbf{r}) = -n_I(\mathbf{r})$$



Christodoulides, et al., Nat. Phys. (2010)



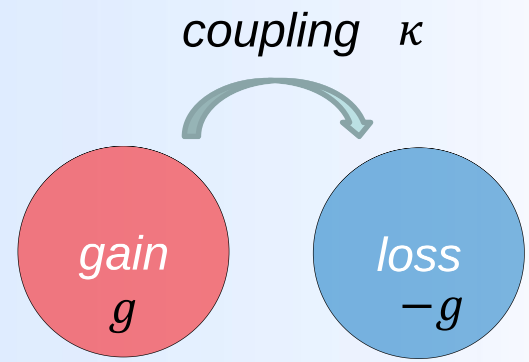


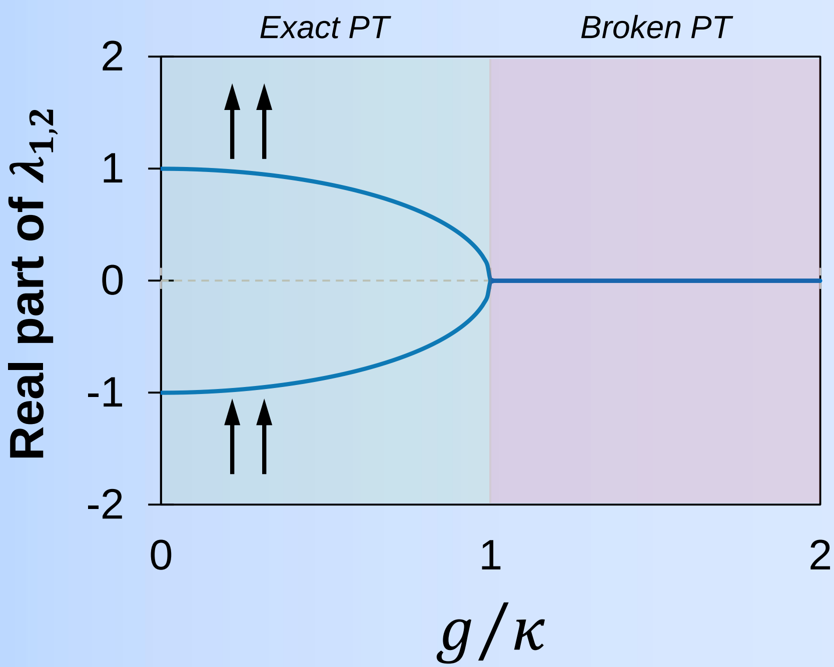


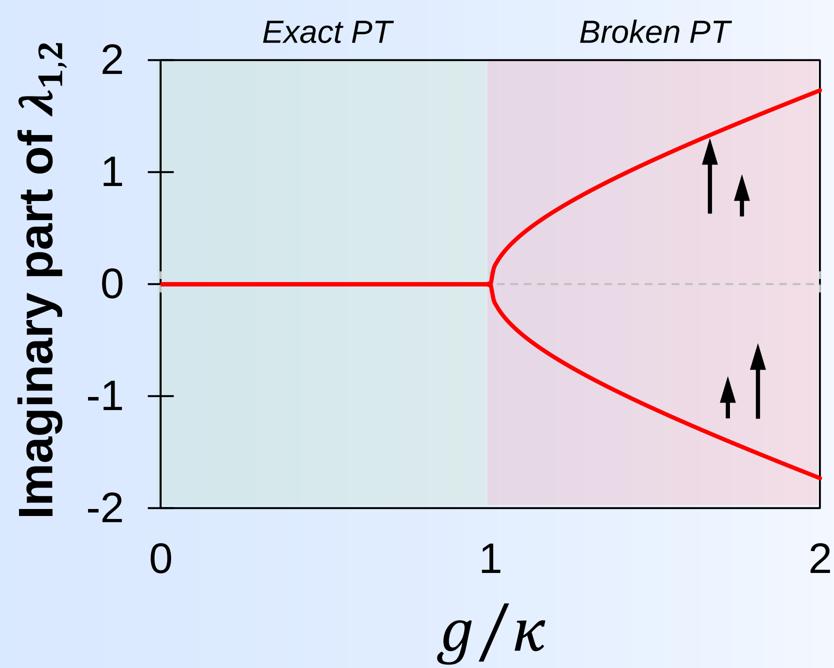
PT-Symmetry in Optics

$$\frac{da}{dz} = i\kappa b + ga$$
$$\frac{db}{dz} = i\kappa a - gb$$

$$\lambda_{1,2} = \sqrt{\kappa^2 - g^2}$$

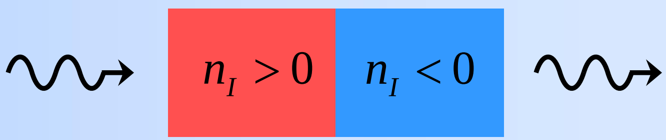


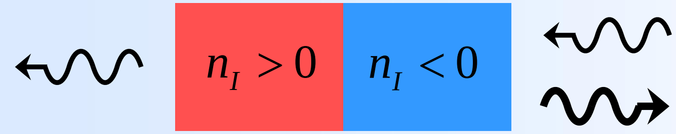




M. A.Miri, A. Alù, *Science* **363**, 42 (2019)

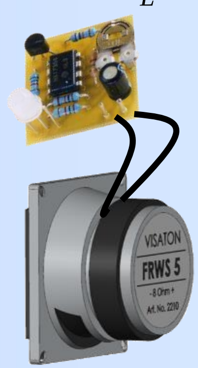
Unidirectional Invisibility



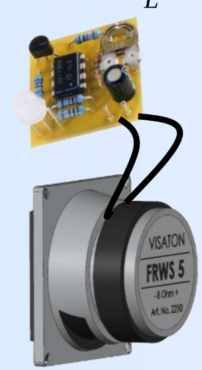


Acoustic PT-symmetry

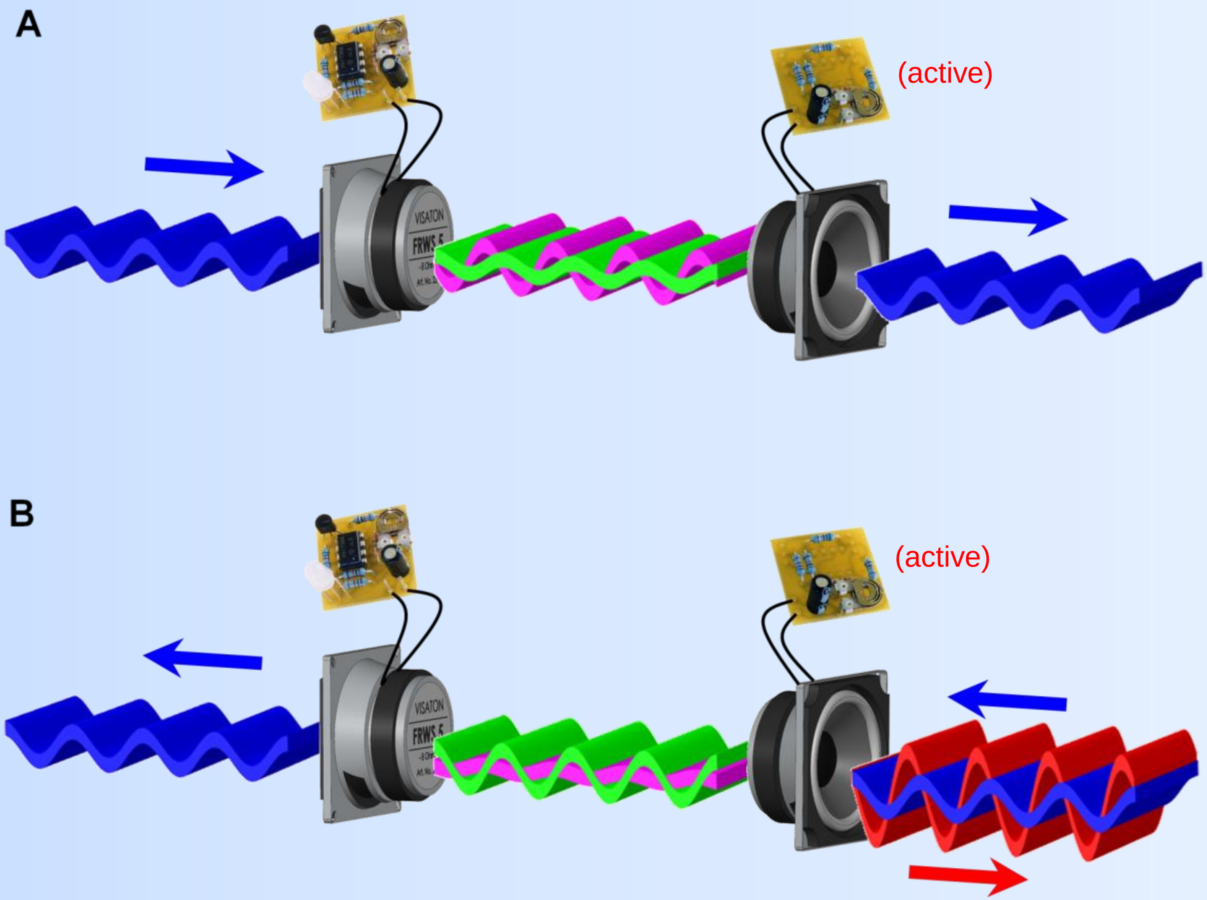
$\mathrm{Re}\, Z_L^{(1)} > 0$



$\mathrm{Re}\, Z_L^{(1)} < 0$ **(active)**

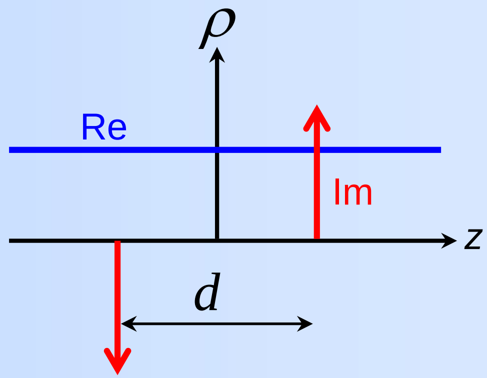


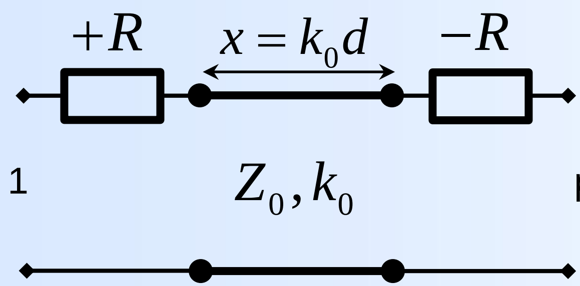
PT-Symmetry for Sound

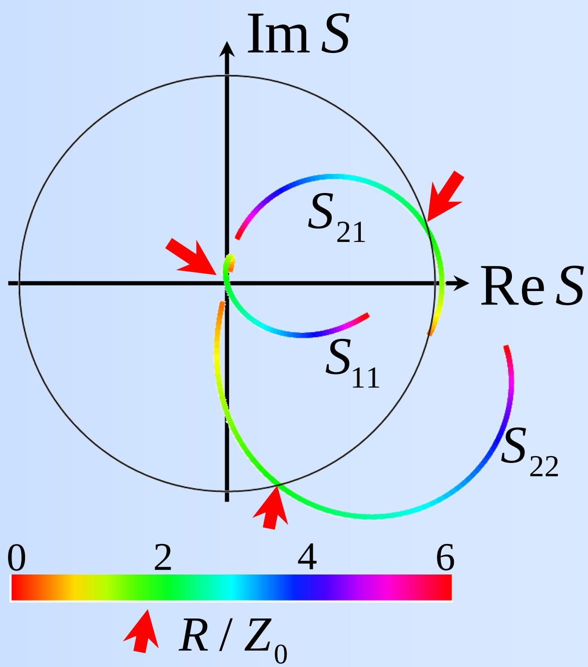


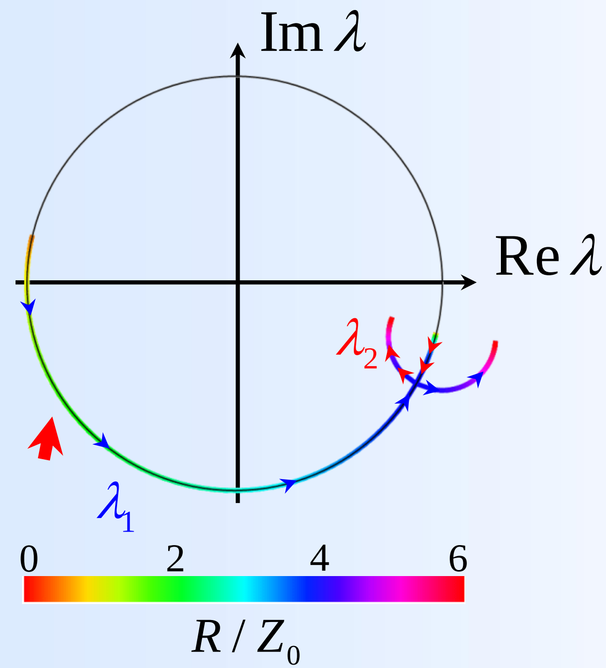
R. Fleury, D. L. Sounas, and A. Alù, *Nat. Comm.* **6**, 5905 (2015)

Scattering Parameters and S-Matrix Eigenvalues

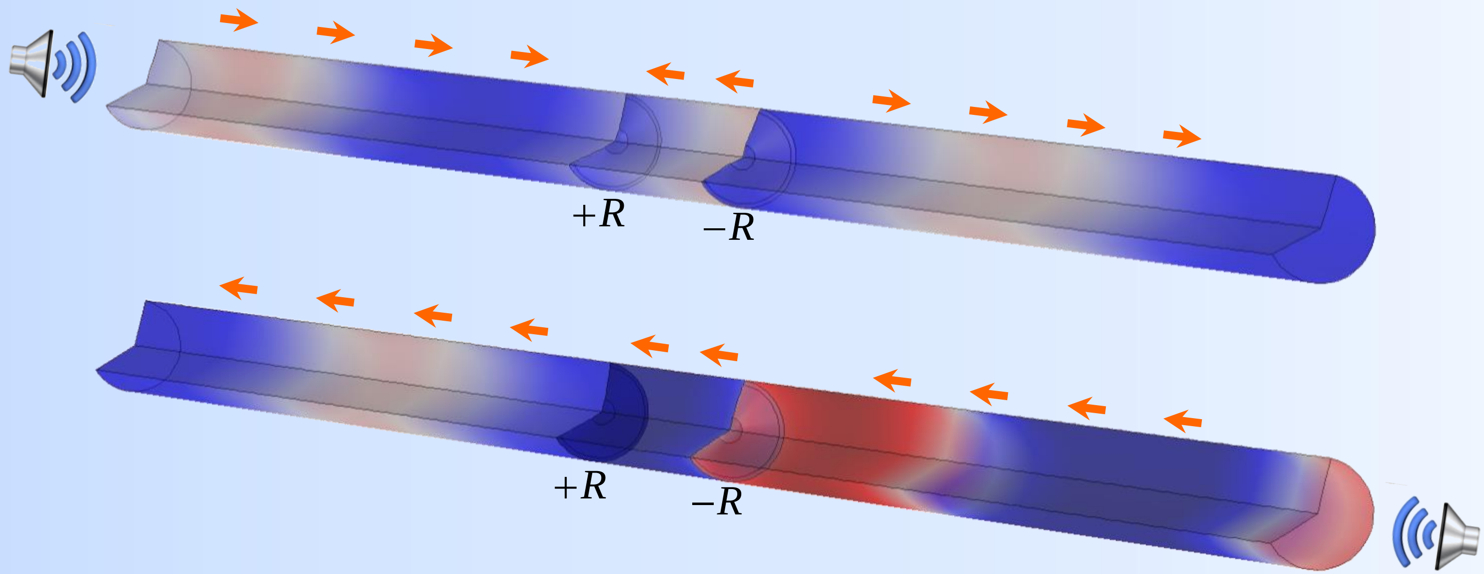








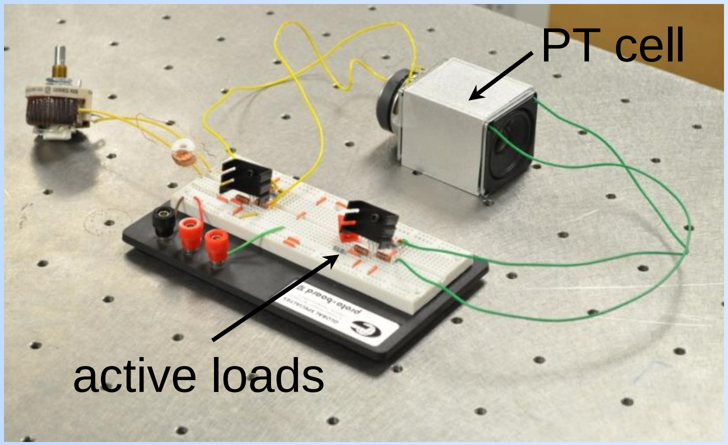
A PT-Symmetric Invisible Acoustic Sensor



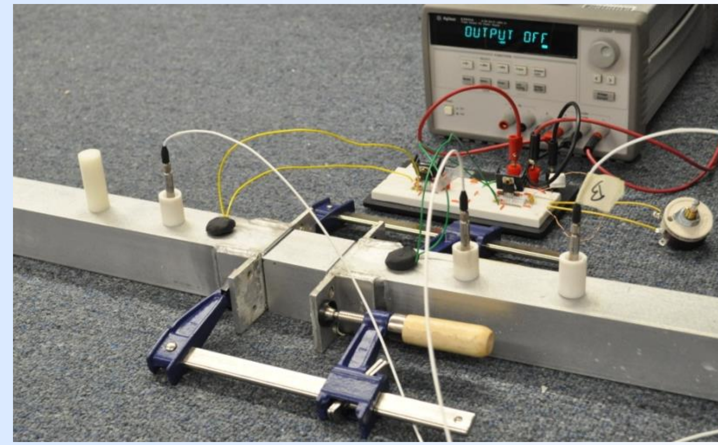
R. Fleury, D. L. Sounas, and A. Alù, *Nat. Comm.* **6**, 5905 (2015)

Experimental Validation

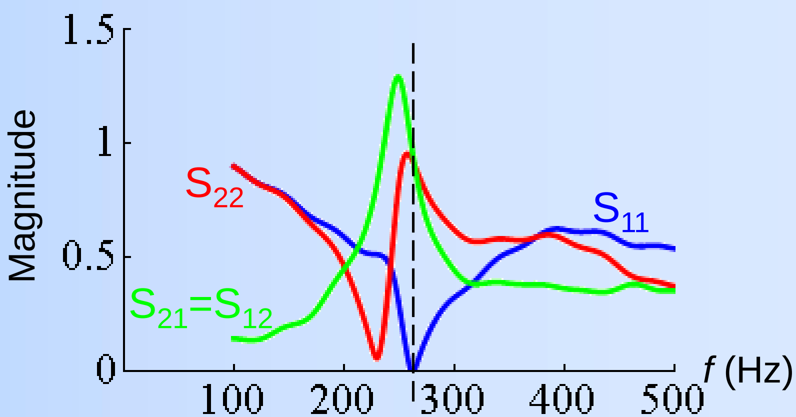
A



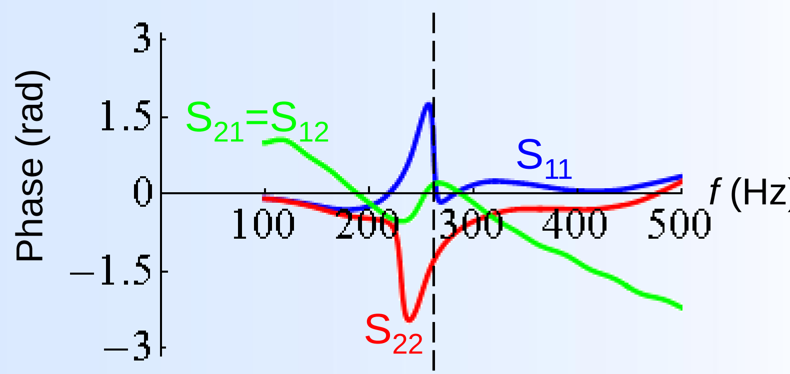
B



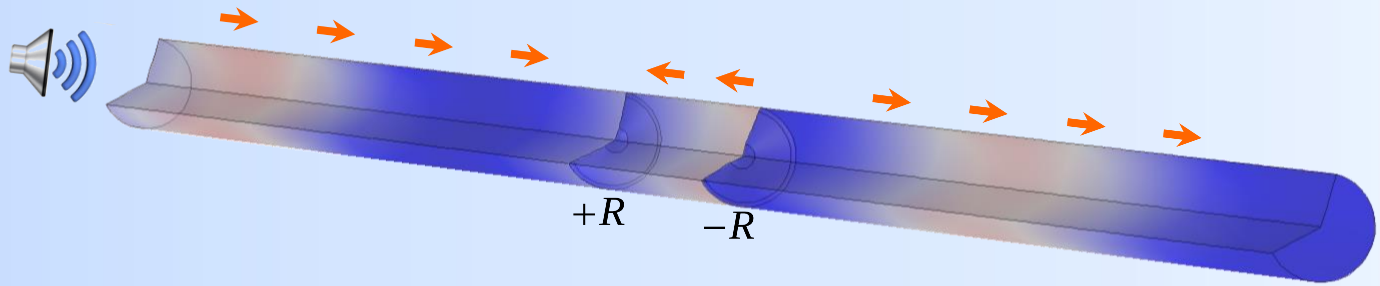
C

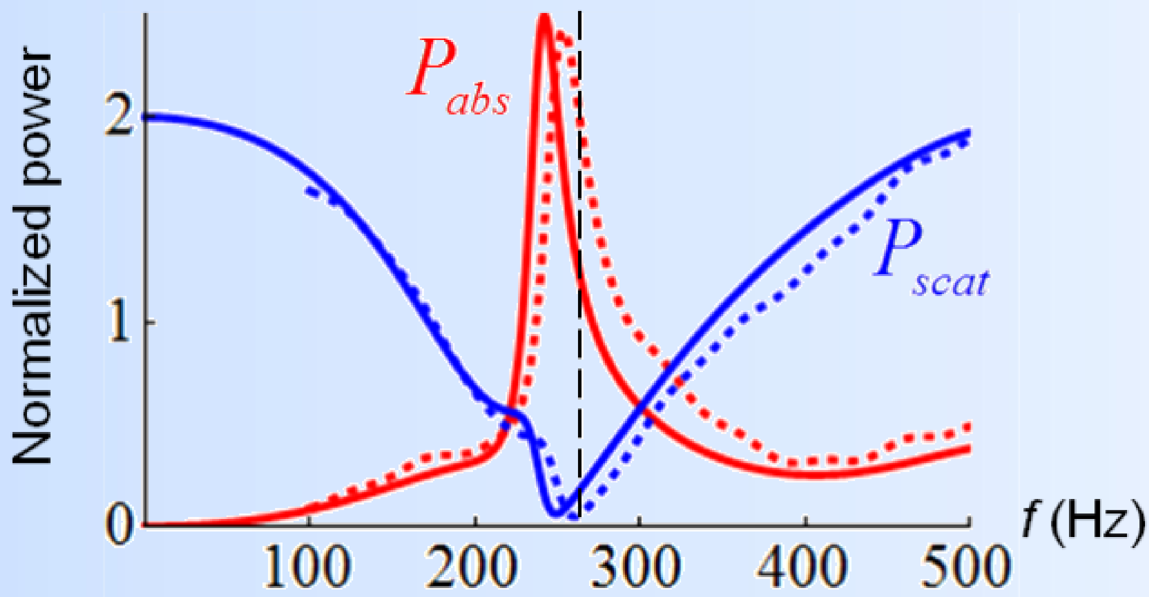


D



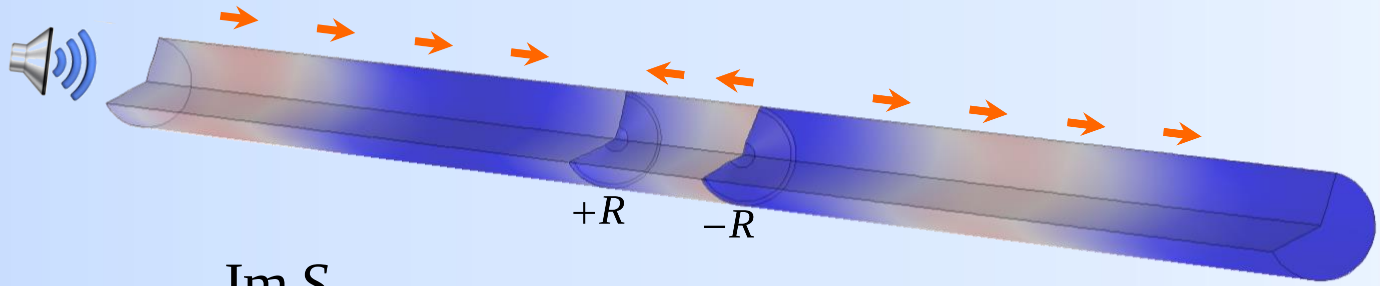
A Sensor That Does Not Cast a Shadow

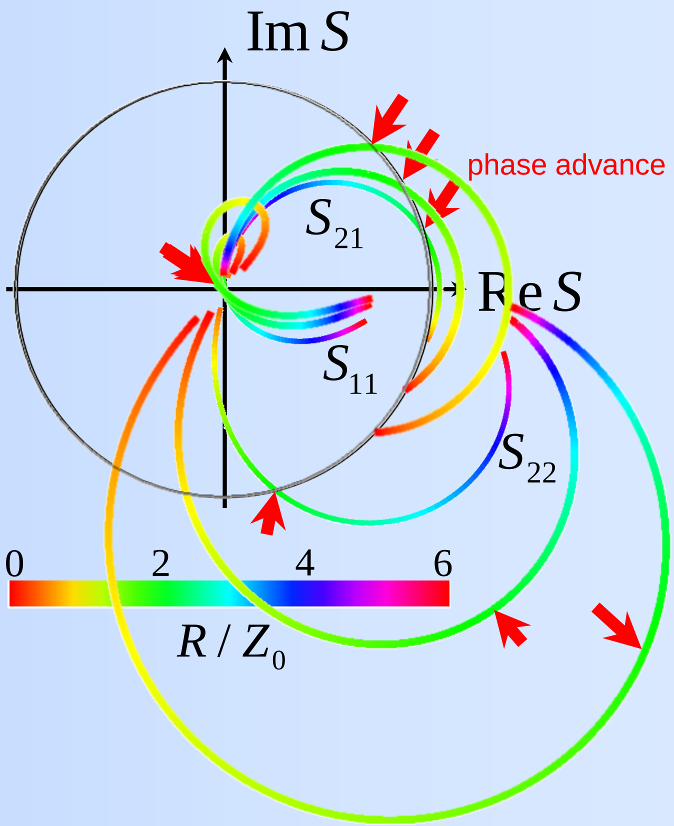




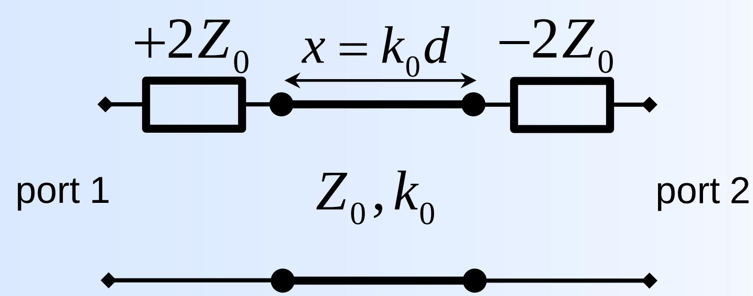
R. Fleury, D. L. Sounas, and A. Alù, *Nat. Comm.* **6**, 5905 (2015)

Loss-Free Negative Refraction

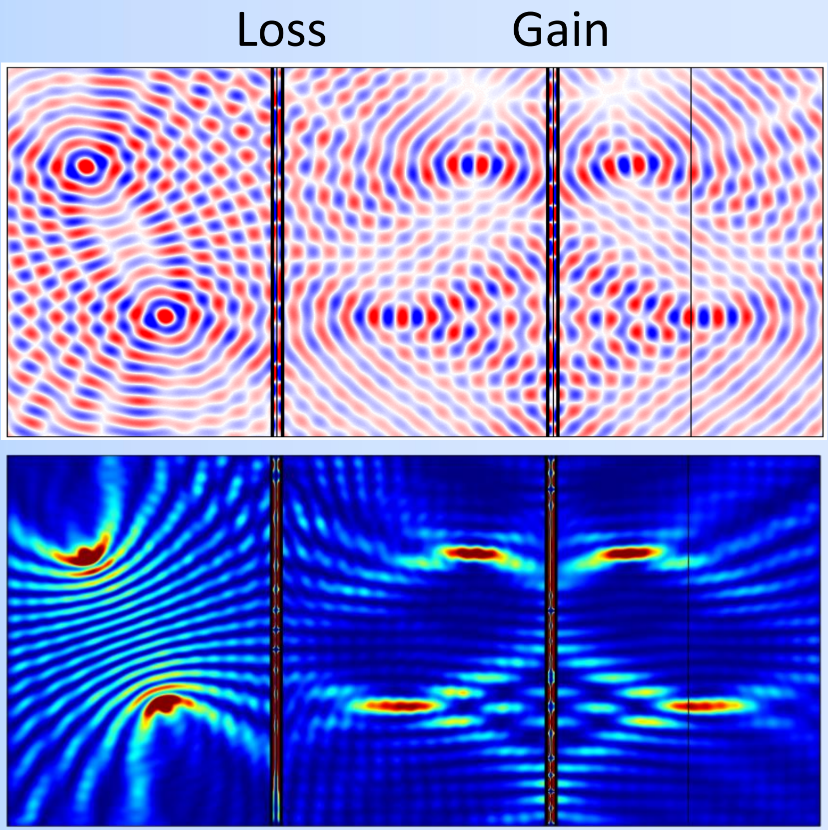


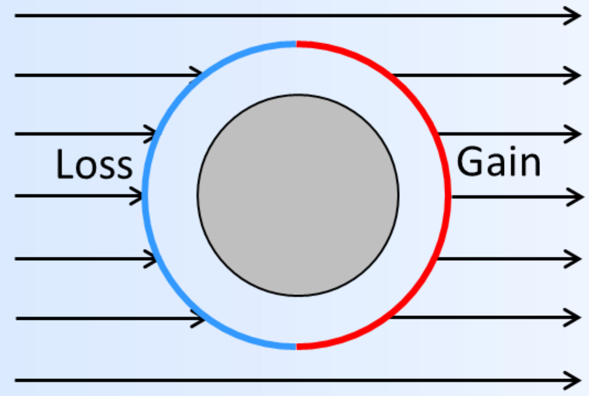


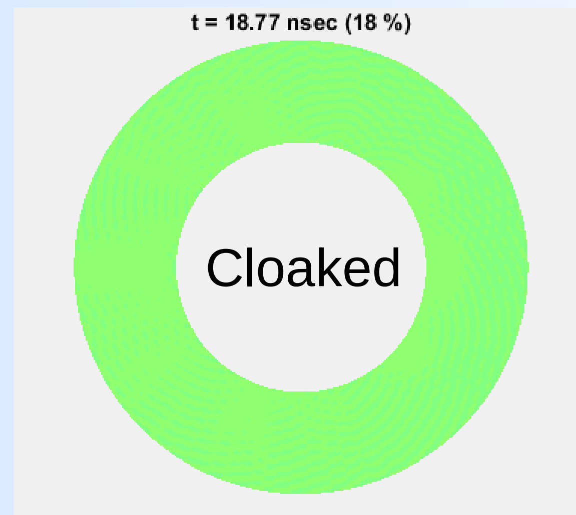
$$S = \begin{pmatrix} 0 & e^{jx} \\ e^{jx} & 2 - 2e^{2jx} \end{pmatrix}$$



Planar Lenses and Cloaks

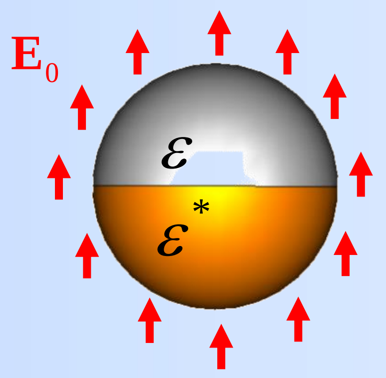


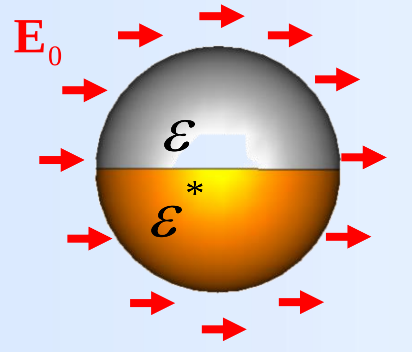




R. Fleury, D. Sounas, and A. Alù, *Phys. Rev. Lett.* **113**, 023903 (2014)
F. Monticone, C. Valagiannopoulos, A. Alù, *Phys. Rev. X* **6**, 041018 (2016)
D. L. Sounas, R. Fleury, and A. Alù, *Phys. Rev. Appl.* **4**, 014005 (2015)

PT Scattering



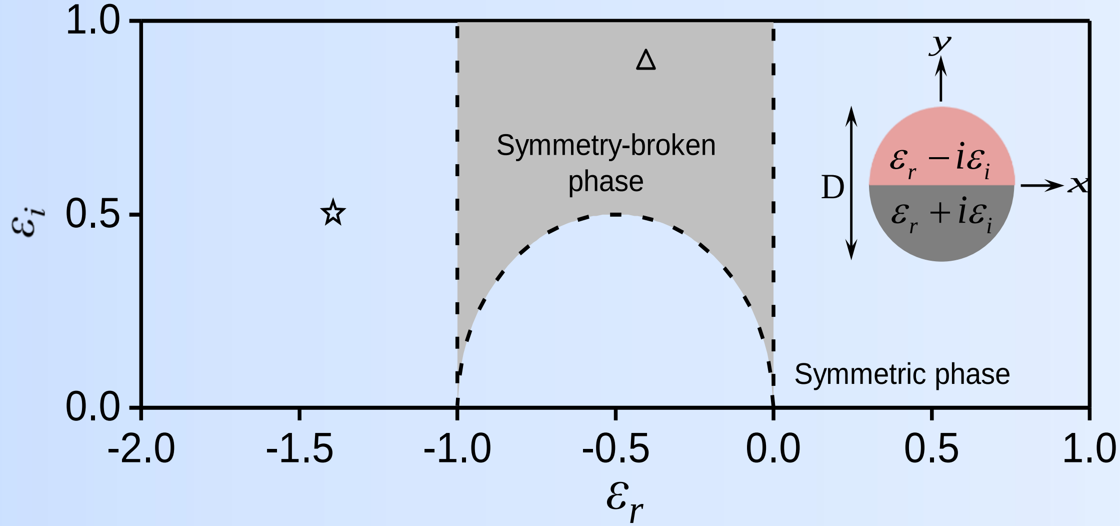


$$\alpha_l = 2 + \frac{8\varepsilon_r\left(3\left(\mathrm{Li}_2\left(\varepsilon^-\right)+\mathrm{Li}_2\left(\varepsilon^+\right)\right)-\pi^2\right)}{3\pi^2\left(\varepsilon_r{}^2+\varepsilon_r+\varepsilon_i{}^2\right)}$$

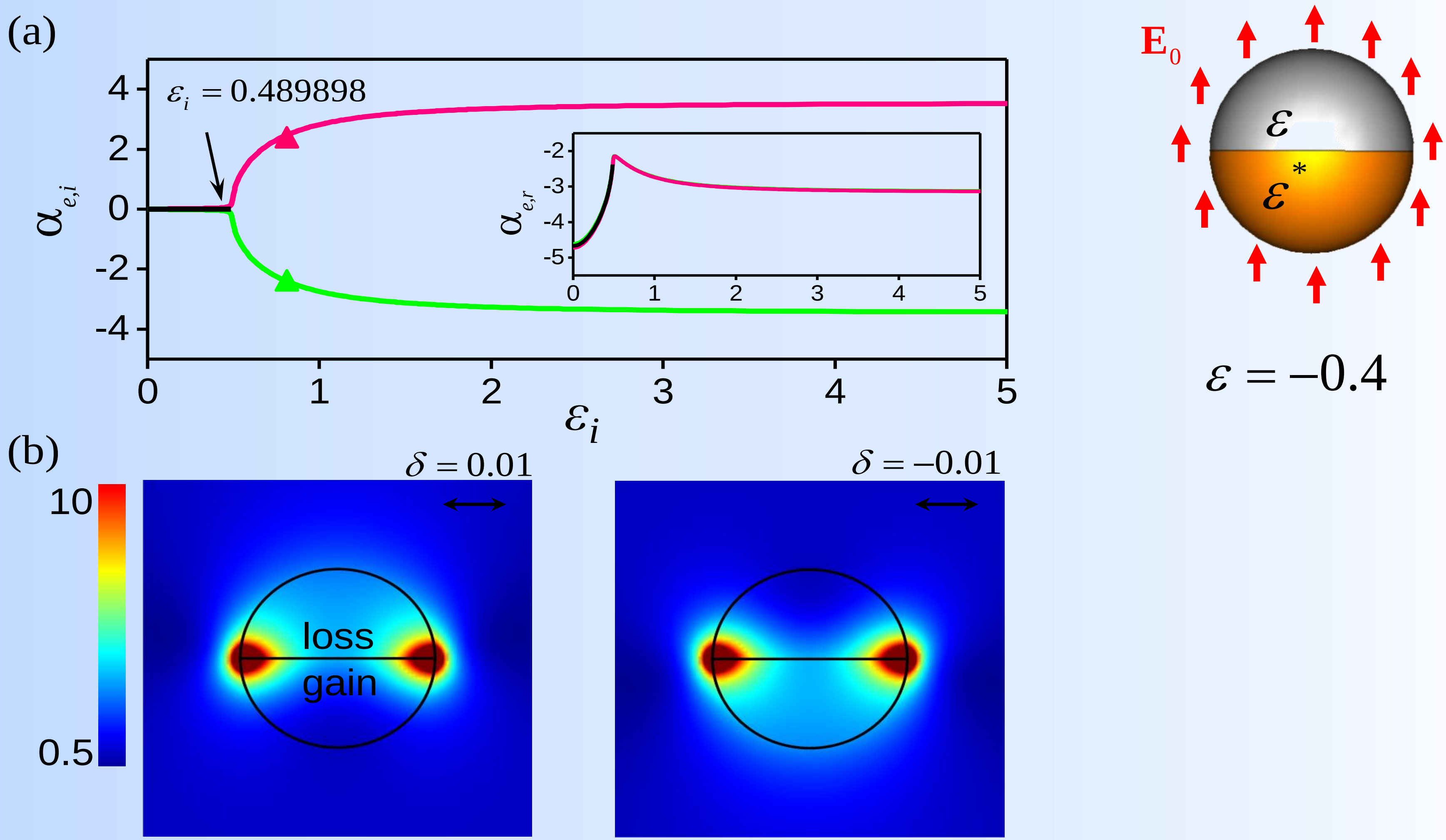
$$\alpha_t = \frac{2\pi^2\left(\varepsilon_r-3\right)-24\varepsilon_r\left(\mathrm{Li}_2\left(\varepsilon^-\right)+\mathrm{Li}_2\left(\varepsilon^+\right)\right)}{3\pi^2\left(\varepsilon_r{}^2+\varepsilon_r+\varepsilon_i{}^2\right)}$$

$$\varepsilon^\pm = \frac{-2\varepsilon_r\left[\left(\varepsilon_r+1\right)^2+\varepsilon_i^2\right]}{\varepsilon_r-i\varepsilon_i\pm\sqrt{2\varepsilon_r^2\left(2+\varepsilon_r\right)+\varepsilon_r+i\varepsilon_i+16\varepsilon_i^4\left(1+\varepsilon_r\right)+2\varepsilon_i^2\left[2+\varepsilon_r\left(9+8\varepsilon_r\left(2+\varepsilon_r\right)\right)\right]}}$$

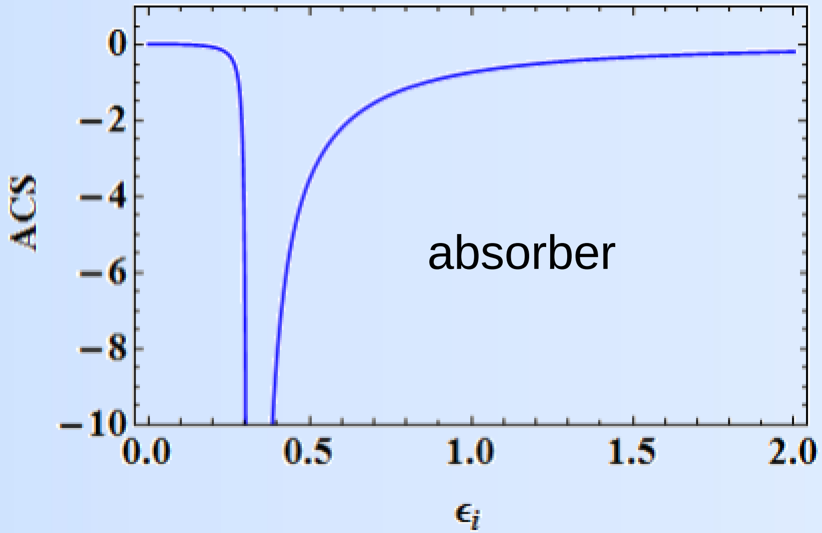
PT Scattering

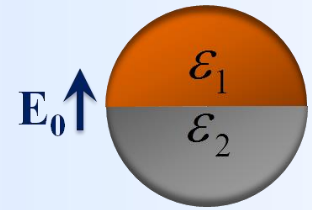


PT Scattering



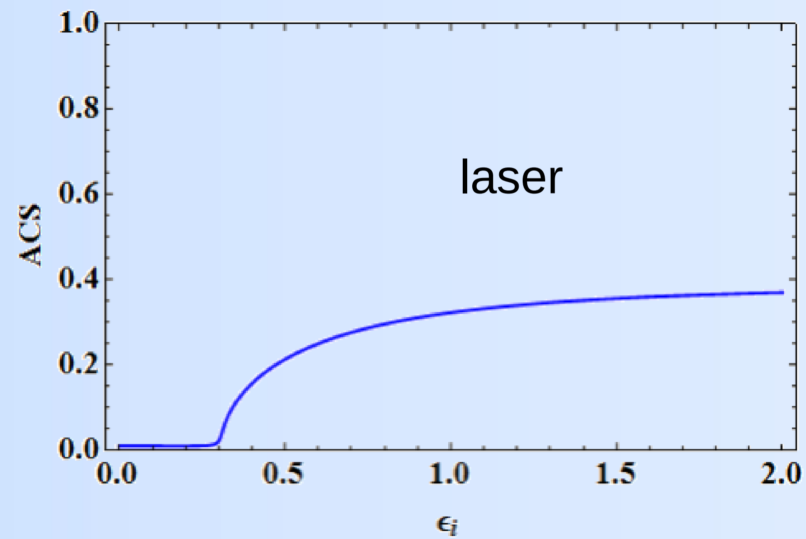
Extreme Anisotropy Beyond the PT Treshold

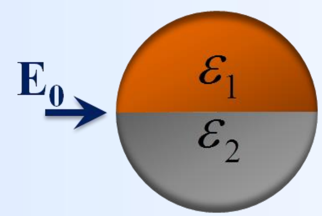




$$\varepsilon_1 = -0.1 - i\varepsilon_i$$

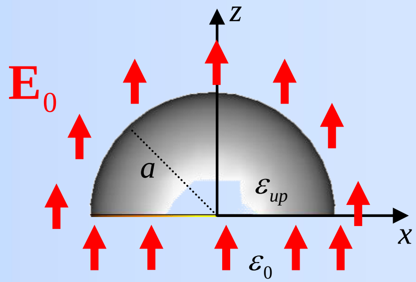
$$\varepsilon_2 = -0.1 + i\varepsilon_i - 0.01i$$

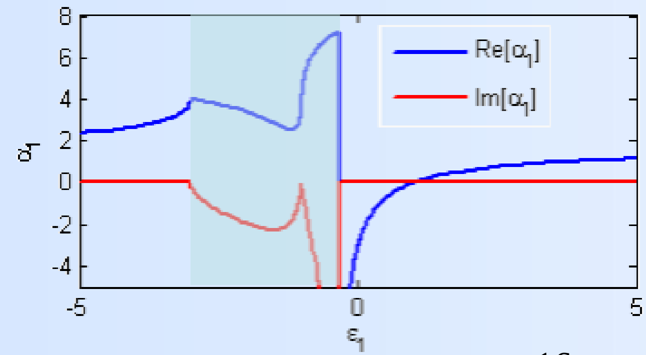




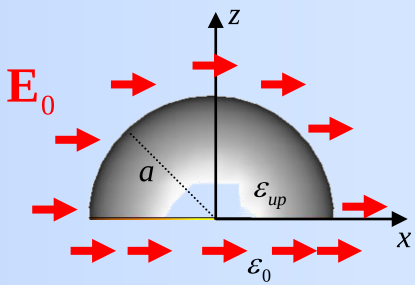
Exact Polarizability in the Resonant Region

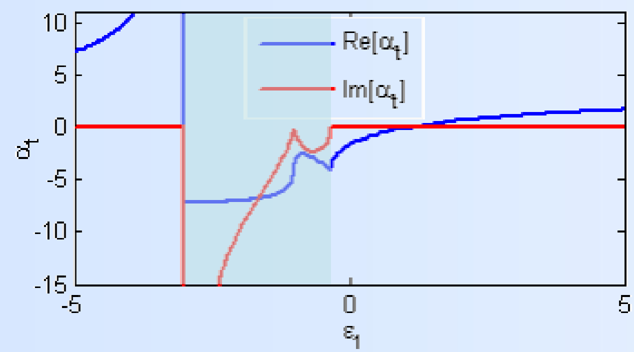
$$\left(-3<\varepsilon_1<-1\right)\vee\left(-1<\varepsilon_1<\frac{-1}{3}\right)$$





$$\mathrm{Im}\left[\alpha_L\right]=\frac{-16}{\pi a^2}\frac{\varepsilon_1+1}{3\varepsilon_1+1}\cosh^{-1}\left(\frac{(\varepsilon_1-1)^2-2(\varepsilon_1+1)^2}{2(\varepsilon_1+1)^2}\right)$$



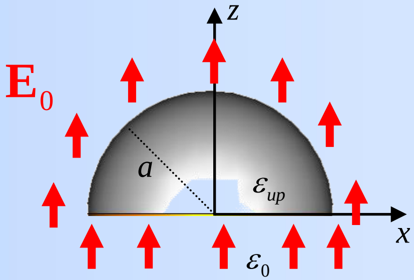


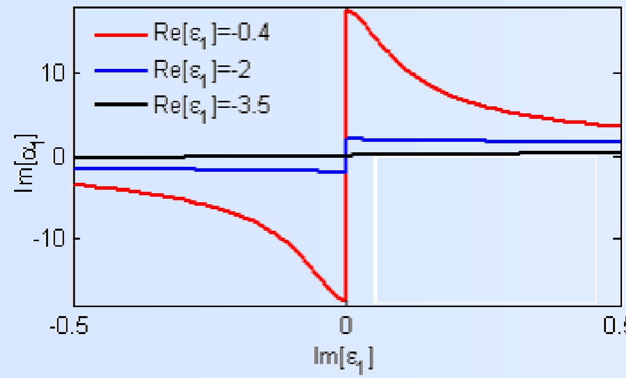
$$\mathrm{Im}[\alpha_T]=\frac{16}{\pi a^2}\frac{\varepsilon_1+1}{\varepsilon_1+3}\cosh^{-1}\left(\frac{(\varepsilon_1-1)^2-2(\varepsilon_1+1)^2}{2(\varepsilon_1+1)^2}\right)\mathrm{sign}\left(\frac{\varepsilon_1-1}{\varepsilon_1+1}\right)$$

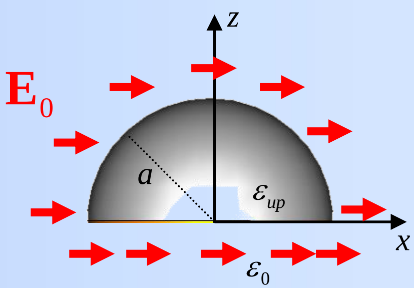
An Absorption Paradox

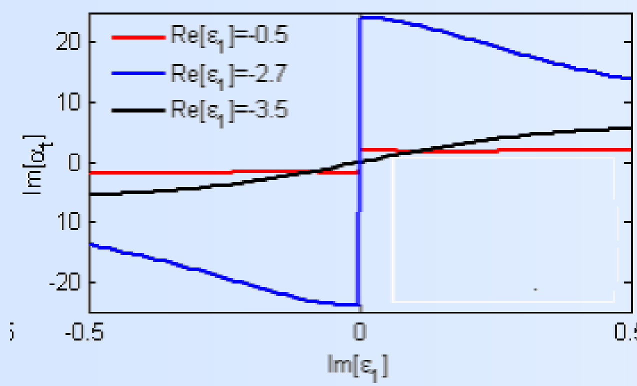
$$P_{ext} = -\omega/2\left|E_0\right|^2 \operatorname{Im}\left[\alpha\right] = P_{abs}$$

$$\left(-3<\varepsilon_1<-1\right) \vee \left(-1<\varepsilon_1<\frac{-1}{3}\right)$$

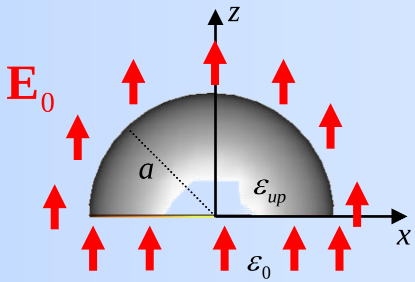








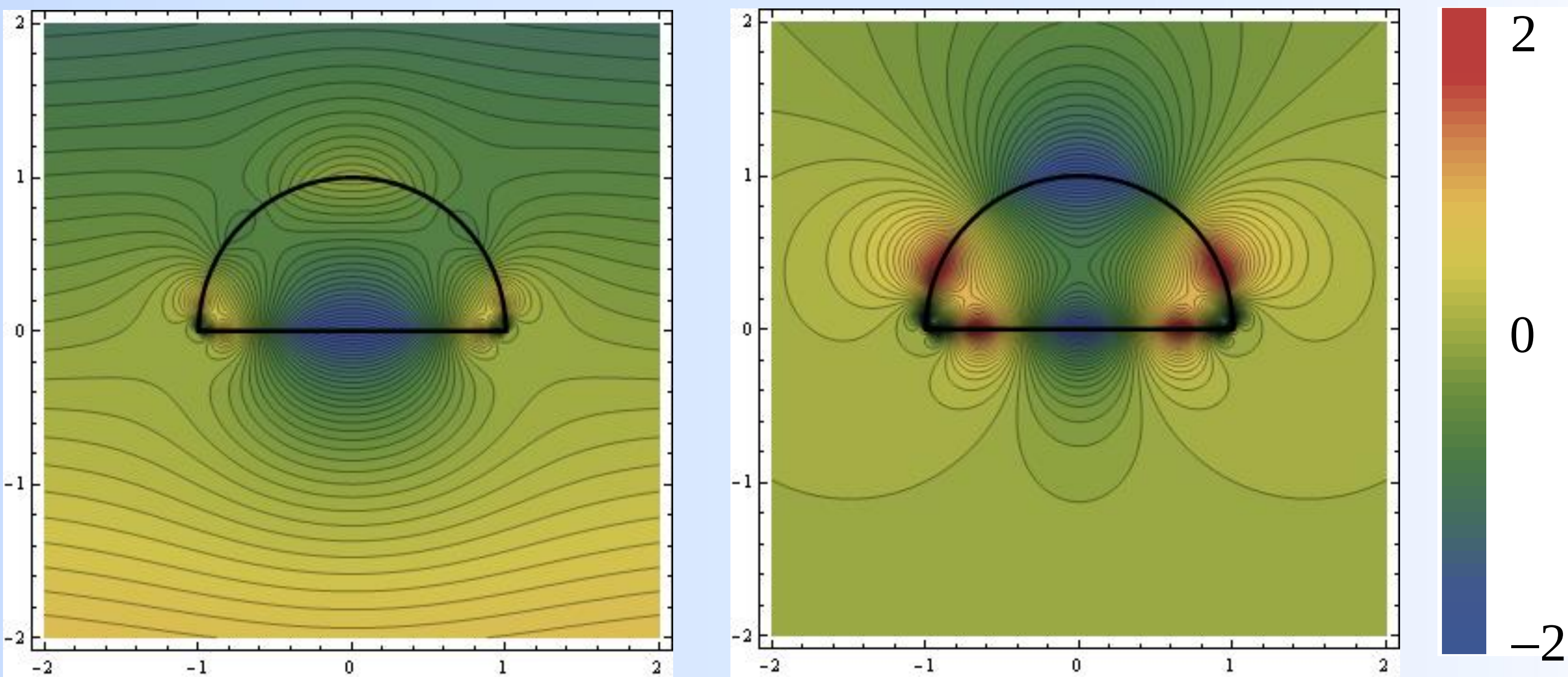
Absorption: Fields in Quadrature with Excitation



$$\mathrm{Im}[\varphi_0] = E_0 \cos(\lambda_0 u)\left[-\frac{\varepsilon_1 - 1}{(\varepsilon_1+1)\sinh(\lambda_0 \pi)}\cosh(\lambda_0 v) - 2\frac{\varepsilon_1+1}{3\varepsilon_1+1}\sinh(\lambda_0 v)\right]$$

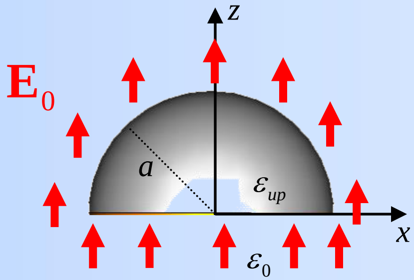
$$\lambda_0 = \frac{1}{\pi}\cosh^{-1}\left(\frac{(\varepsilon_1-1)^2 - 2(\varepsilon_1+1)^2}{2(\varepsilon_1+1)^2}\right)$$

$\varepsilon_1 = -1.1$



$\mathrm{Re}[\varphi]$ $\mathrm{Im}[\varphi]$

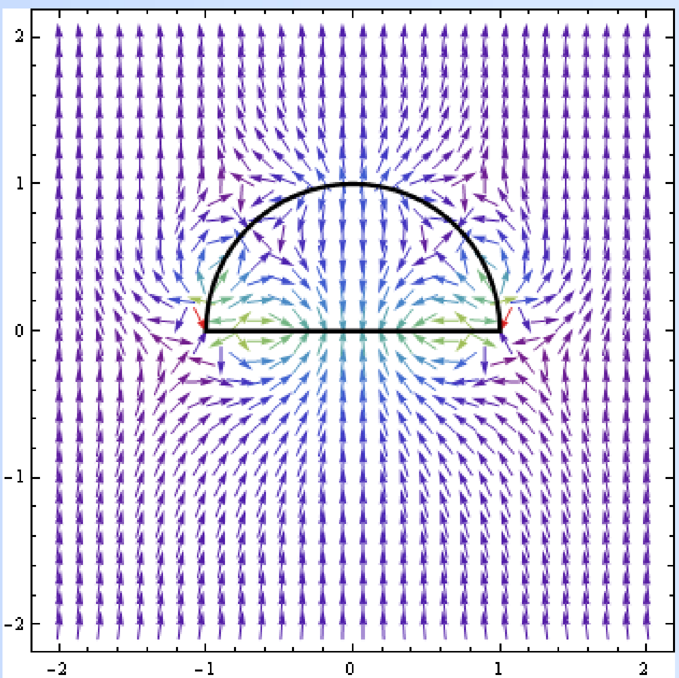
Absorption: Fields in Quadrature with Excitation



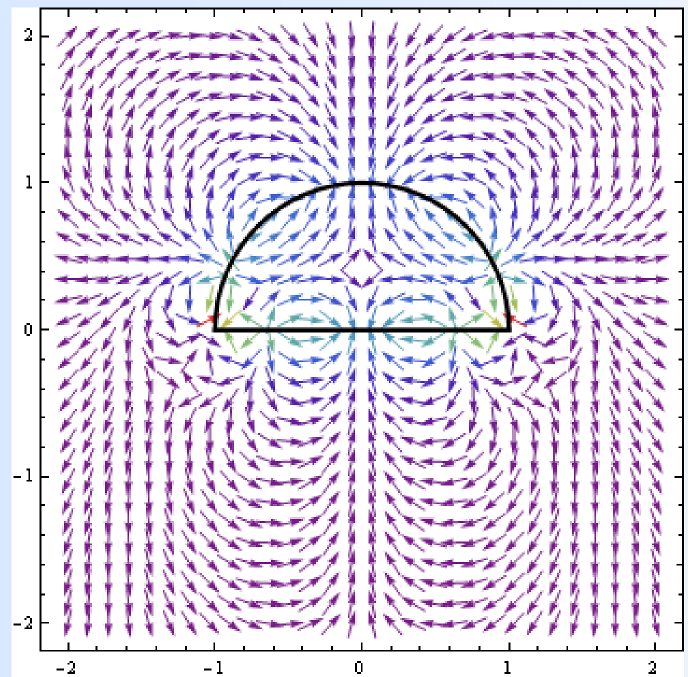
$$\mathrm{Im}[\varphi_0] = E_0 \cos(\lambda_0 u)\left[-\frac{\varepsilon_1 - 1}{(\varepsilon_1+1)\sinh(\lambda_0 \pi)}\cosh(\lambda_0 v) - 2\frac{\varepsilon_1+1}{3\varepsilon_1+1}\sinh(\lambda_0 v)\right]$$

$$\lambda_0 = \frac{1}{\pi}\cosh^{-1}\left(\frac{(\varepsilon_1-1)^2 - 2(\varepsilon_1+1)^2}{2(\varepsilon_1+1)^2}\right)$$

$\varepsilon_1 = -1.1$

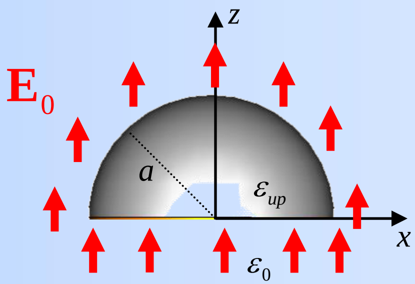


Re[**E**]



Im[**E**]

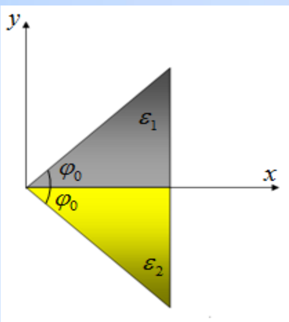
Absorption: Fields in Quadrature with Excitation

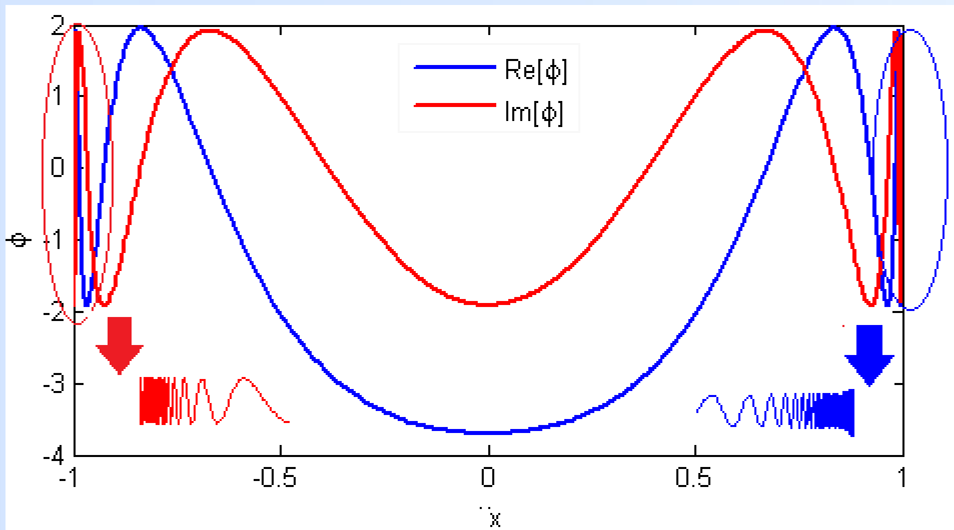


$$\mathrm{Im}[\varphi_0] = E_0 \cos(\lambda_0 u)\left[-\frac{\varepsilon_1 - 1}{(\varepsilon_1+1)\sinh(\lambda_0 \pi)}\cosh(\lambda_0 v) - 2\frac{\varepsilon_1+1}{3\varepsilon_1+1}\sinh(\lambda_0 v)\right]$$

$$\lambda_0 = \frac{1}{\pi}\cosh^{-1}\left(\frac{(\varepsilon_1-1)^2 - 2(\varepsilon_1+1)^2}{2(\varepsilon_1+1)^2}\right)$$

$\varepsilon_1 = -1.1$





Acknowledgements