Work – Energy Theorem: $W_{net} = \Delta KE$

Object: To verify the Work-Energy Theorem by comparing the net work done on an object to its change in Kinetic Energy.

Apparatus: Computer, PASCO Signal Interface Box and *DataStudio* software, end-stop, track, force sensor, mass hanger and mass set, smart pulley, clamp (for smart pulley) dynamics cart, and 1.2-meter track.

Foreword

If an object of mass "m" experiences a net force F_{net} **that is parallel to the motion** over a distance "d," the net work done is:

(1)
$$W_{net} = F_{net}d$$

If the net work done on the object changes the object's speed, the object's kinetic energy changes as follows:

(2)
$$W_{net} = \Delta KE = KE_f - KE_i = mv_f^2/2 - mv_i^2/2$$

where W_{net} is the net work done on the object, v_f is the final speed of the object, and v_i is the initial speed of the object. Equation (2) is a statement of the Work-Energy Theorem, namely that the net work done on the object is equal to its change in Kinetic Energy.
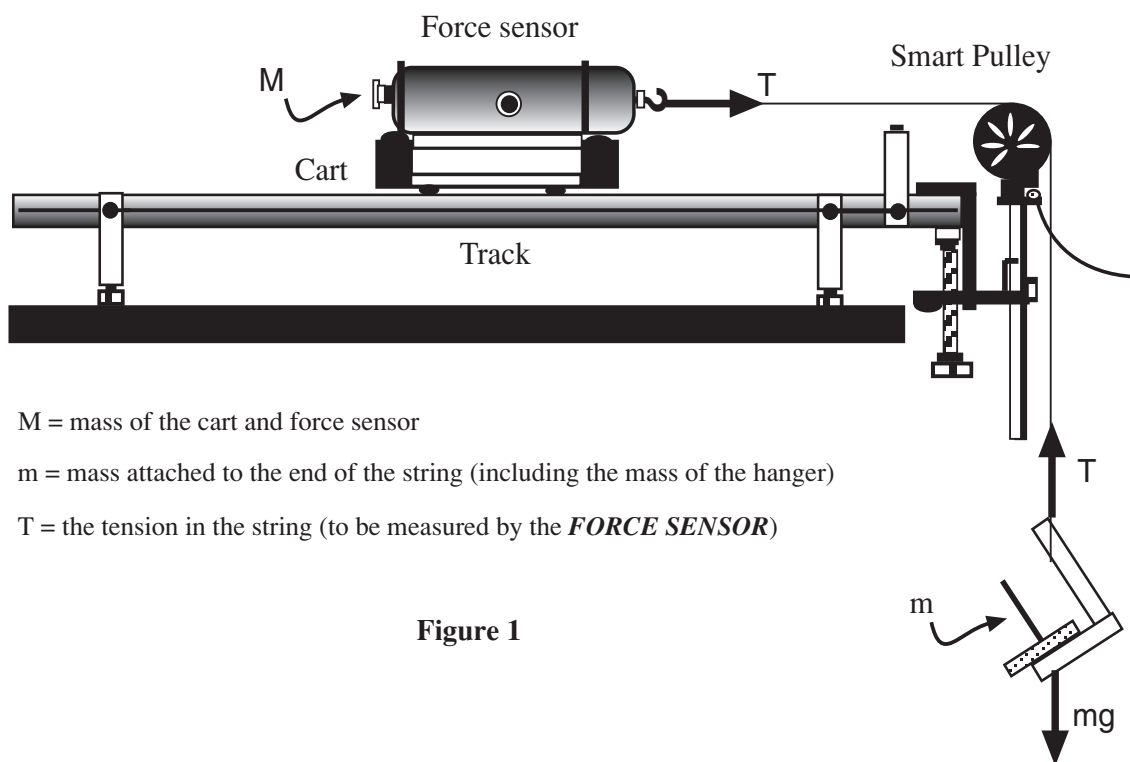
M = mass of the cart and force sensor

m = mass attached to the end of the string (including the mass of the hanger)

T = the tension in the string (to be measured by the *FORCE SENSOR*)

Figure 1

Let's apply the Work-Energy Theorem to the situation shown in Figure 1. Let M equal the mass of the dynamic cart and the force sensor, m equal the mass (including the weight hanger) attached to the end of the string, and T equal the Tension in the string (note that this Tension is measured by the force sensor). Let's agree to ignore friction and the mass of the pulley.

First, let's apply the Work-Energy Theorem to M (the cart and the force sensor).

The net work done on M is:

(3) $$W_{net} = Td$$

Where d is the distance that the force T (Tension) acts.

The change in the Kinetic Energy of M is:

(4) $$\Delta KE = \frac{1}{2}Mv_f^2 - \frac{1}{2}Mv_i^2$$

In this experiment, we will determine and compare the net work done on M to its change in Kinetic Energy.

Also, let's apply the Work-Energy Theorem to the entire system, that is M + m (the total mass of the system).

The net work done on M + m is:

(5) $$W_{net} = F_{net}d = mgd$$

The change in the Kinetic Energy of M + m is:

(6) $$\Delta KE = \frac{1}{2}(M + m)v_f^2 - \frac{1}{2}(M + m)v_i^2$$

In this experiment, we will determine and compare the net work done on (M+ m) to the change in Kinetic Energy of (M + m).

Of course, we could just as easily apply the same principles to the small mass at the end (m) of the string individually, but due to time restrictions we will not incorporate that into this experiment.

Finally, since the only force acting on the system (M + m) is conservative, the total energy of the system is conserved. As the system moves, the kinetic energy increases, the gravitational potential energy of the system decreases, but the total energy remains constant. Another way of saying this is that the change in the kinetic energy plus the change in potential energy is zero. For the purpose of determining the gravitational potential energy, let's agree to select zero gravitational energy to be at the floor. This may be written as shown below:

The change in potential energy of the system is:

(7) $$\Delta U = U_f - U_i = mg(h - d) - mgh = -mgd$$

5M–2

Where h is the initial distance of the mass m from the floor, d is the distance the system has moved at any position (as determined by the smart pulley), and (h - d) is the distance of the mass m above the zero gravitational energy position (the floor).

The change in kinetic energy of the system at any position is:

(8)
$$KE = KE_f - KE_i = \frac{1}{2}(M + m)v^2 - 0 = \frac{1}{2}(M + m)v^2$$

The total change in Energy of the system at any position is:

(9)
$$\Delta E = \Delta KE + \Delta U = \frac{1}{2}(M + m)v^2 - mgd$$

Procedure

Part I. Computer Setup

1.   Connect the Interface Box to the computer, turn on the interface, and turn on the computer.

2.   Connect the force sensor's DIN plug into Analog Channel A of the interface.

3.   Connect the Smart Pulley's stereo phone plug into Digital Channel 1.

Part II. Equipment Setup — refer to Figure 1

1.   Put the cart near the middle of the track and level the track by raising or lowering one end so the cart will not roll one way or the other.

2.   Put one end-stop at the right end of the track. Place the cart next to the end stop.

3.   Put a clamp on the end of the track. Mount the smart pulley's rod in the table clamp so that the groove in the pulley is the same height as the hook on the force sensor mounted on the cart.

4.   Use a piece of string that is about 10 centimeters longer than the distance from the top of the smart pulley to the floor. Connect one end of a string to the force sensor's hook. Place the string in the smart pulley's groove.

5.   Attach the mass holder to the end of the string so that the bottom of the holder is just above the floor when the end of the cart is against the end-stop.

6.   Let's agree to use a hanging mass of 0.050kg (including the mass hanger). This will cause a maximum Force of 0.49N.   *__m = 0.050kg__*

7.   Also, let's agree to add 1.00kg to the cart so that M= mass of cart and force sensor + 1.00 kg. Measure and record the mass of the cart with the force sensor.

<div align="center">

__M = mass of cart with force sensor +1.00kg = _____

</div>

Part III. Software Setup

1.   Open the *DataStudio* software by double clicking on the icon. If asked what action to take at this time, respond by clicking on the "*Create Experiment*" icon.

2.   **SMART PULLEY**
    a. **Connect**—In the *Experiment Setup* window; tell the software you have plugged a smart pulley into Digital Channel 1. This is done by clicking and dragging the *Smart Pulley* icon from the list of sensors to Digital Channel 1. At this point you will notice a smart pulley icon connected to Digital Channel 1 of the interface window.

    b. **Setup**—The software keeps track of the time between spoke interruptions. By knowing the *"spoke angle spacing"* and *"spoke arc length"* the computer can keep track of up to eleven different measurements. By default, the software only keeps track of the velocity, but in this experiment we are also interested in the distance traveled. To modify this:

        i.   Double click on the smart pulley icon to bring up sensor properties.

        ii.   Click on the tab labeled "*Measurement*" and *add a check* beside *"POSITION."*

        iii.   Click "**OK**" —you should now see an icon for **Position** and **Velocity** in the "**DATA**" Window in the upper left corner of the screen.

3.   **FORCE SENSOR**
    a. **Connect** —Much like what you did with the smart pulley, click and drag the "*Force Sensor*" icon from the list of sensors to ANALOG channel A. A *Force Sensor* icon should be "connected" to analog channel A.

    b. **Zero the Sensor**—Make sure there are no Forces acting on the Force Sensor and press the **TARE** button located on the top or the side of the sensor.

    c. **Setup**—Double-click the Force Sensor icon to bring up the "*Sensor Properties*" menu.

        i.   Under the "*General*" tab, modify the "*Sample Rate*" field to 50 Hz by clicking the (+) button twice.

        ii.   Click on the "*Calibration*" tab and modify the "*Sensitivity*" to *HIGH(100x)*.

iii. Above that, still under the "**Calibration**" tab, there are two areas labeled "**High Point**" and "**Low Point**." In this area, do NOT modify the fields for the Voltages, but **change** the field for "**High Point Value**" to have the negative (**–50.00N**) and **change** the field for "**Low Point Value**" to be positive (**50.00N**). *As a result of this, a pulling force will now be positive instead of negative*.

iv. Click **OK**

4. When the cart hits the end stop, the Force will rapidly increase and cause an area where the data is hard to analyze. To avoid this, you will set a **stop condition** so that the computer stops taking data before this happens. To do this:

    a. Click on the "**Options**" button," ![Options] on the "**Experiment Setup**" window to bring up a window labeled "**Sampling Options**."

    b. Click on the tab labelled "**Automatic Stop**," check the box beside "**Data Measurement**."

    c. In the fields that become active, set "**Position, Ch1 (m)…Is Above…0.40m**." Therefore, the computer will stop taking data after the cart has traveled 0.40m. "*Make sure the cart has a range of motion of about 0.45m or more*.

    d. Click **OK**

Part IV. Set up the Calculations

1. Click on the Calculator Icon in the Main Toolbar. ![Calculate]

2. In order to prevent having to numerically enter the masses for multiple calculations, enter the two masses (M and m) as experimental constants. You may also *enter gravity* (g = 9.81 m/s/s). Refer to the section of the appendix labeled "**Entering Experimental Constants**" or use the HELP feature of *DataStudio (search for Experiment Constants)* if you need assistance.

3. Using the calculator, **perform the following calculations**: (1) Kinetic Energy of M, (2) Kinetic Energy of System, (3) Net Force on System *(F_{net}= mg – T + T, where T is the measurement of the Force Sensor)*, and the (4) Total Change in Energy of the System. Refer to the section of the appendix labeled "**Using the Calculator**" or use the HELP feature of *DataStudio* if you need assistance.

    • Using the "**Properties**" button, be sure to
        i. Give each calculation units

        ii. Modify the "**Precision**" (number of decimal places) of each calculation to **four (4)**.

Part V. Data Recording

1.   Pull the cart away from the end-stop so the mass holder on the end of the string is just below the smart pulley. Make sure it can travel at least 0.45m before hitting the end-stop.

2.   Turn the pulley so the beam of the smart pulley photogate is not blocked (the light-emitting diode or LED on top of the photogate is not lit).

3.   Click the *"Start"* button to begin data recording.

4.   Release the cart so that it can move toward the Smart Pulley. The computer should automatically stop after the cart has traveled 0.40m. If done correctly, you will not have to hit the stop button.

Part VI. Analyzing the Data—Graphs

You should produce 3 graph displays.

Graph Display #1–Work-Energy Theorem applied to M

1.   This graph should give you the ability to compare the Net Work done on the cart by the Tension in the string to the change in Kinetic Energy over the same distance. Therefore, on top, it should have a graph of Tension vs. Position and on the same display under that graph should be a graph of Kinetic Energy of M vs. Position.

2.   Drag the "*Graph*" icon from the DISPLAYS window to the icon for the Force Sensor and release. This will generate a graph of Force vs. Time.

3.   To change the X-Axis from Time to position, drag the "*Position*" icon from the DATA window to the X-Axis until it makes a dotted box around the X-Axis and release. You should now have a graph of Force vs. Position.

4.   To add the graph of Kinetic Energy of M, drag the appropriate calculator icon *(the one that calculates the Kinetic Energy of M)* from the DATA window to the graph you just made (not on top of an axis) and release. This should create a graph of KE of M vs. time graph in the same window as the Force vs. Position graph.

5.   Change the X-Axis of the Kinetic Energy graph to Position just like you did the for the Force graph in step 3. Once you have an X-Axis for position on both graphs, click the "*Align Matching X-Scales*" button, and this will keep the X-Axes for the two graphs locked.

6.   On the Force vs. Position graph, select a data range and highlight it by using the mouse to drag a box around the data points. Under the Statistics pull down menu (Σ) select "*Area*," and the computer will integrate (*calculate the area under*) that region of the graph. This area will represent the Total Force Applied multiplied by the Total Distance which it was applied over…*a.k.a. the NET WORK*.

7. On the Kinetic Energy vs. Position graph, we want to find the change in Kinetic Energy that corresponds to the same distance that we just integrated over. To do this, *DataStudio* comes with very useful utilities called the "*Smart Tool*" and the "*Delta Tool*." Use the smart tool and delta tool to find the increase in Kinetic Energy over this range. If you're not familiar with the smart tool, refer to the section of the Appendix labeled "**Using the Smart Tool and Delta Tool**" or the HELP feature of *DataStudio*.

8. Print the graph display and calculate the percent difference between the Net Work as calculated from the F vs Position graph and the change in KE from the KE graph. *Account for any differences*.

Graph Display #2–Work-Energy Theorem Applied to Entire System

1. Generate a graph display that will allow you to compare the Net Work for the SYSTEM to the change in Kinetic Energy of the SYSTEM much like you just did for studying "M."

2. Analyze the data in the same fashion and print this graph display (should contain 2 graphs).

3. Calculate a percent difference, and **account for the differences**.

Graph Display #3–Change in Energy of the System.

1. Create a graph of Total Change in Energy of the System vs. Position.

2. Select an appropriate range for the axes (see "**Manual Scaling of Graph Axes**" in the appendix if needed)

3. Print this graph and make appropriate concluding remarks (*what you expected vs. what the graph shows*) on the plot.

Your report should consist of an abstract and all four graph printouts with their remarks and comments.

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Purpose:

Conclusions from Data Analysis: (Explain how you analyzed the data you collected and related the results to the physical concept under investigation.)

Explanation of Error: (Include a Percent Difference or Percent Error calculation as appropriate. List reasons for your error that make sense based on your error; i.e. if you calculate a theoretical velocity that is lower than your experimental velocity, friction cannot be the cause for your error.)